GPS/GLONASS Attitude Determination with a Common Clock using a Single Difference Approach

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BIOGRAPHY

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ABSTRACT

This paper presents a novel approach to combining GPS and GLONASS to estimate heading and pitch using a twin GPS/GLONASS receiver system and a single difference approach. The theory behind the use of GPS for attitude component determination is well known. The augmentation is however not straightforward because GLONASS employs the technique of frequency division multiple access (FDMA) to distinguish the signals from different satellites, which makes the double differenced (DD) carrier phase observables no longer practical. The use of between-receiver single differenced (SD) carrier phase observables is used herein to deal with this problem. In this case however, receiver clocks and other errors do not cancel out. An external clock serves as a common oscillator for the two receivers to reduce these errors. Remaining time and other biases are estimated using a low pass filter. The characteristics of the remaining errors, including multipath, are carefully studied. The single differenced ambiguities can then be resolved and the heading and pitch can be determined with an accuracy similar to that of GPS under good visibility conditions. Under reduced visibility, the combined GPS/GLONASS approach however yields superior availability. The results of selected static and kinematic tests conducted with a pair of GPS/GLONASS receivers are described to demonstrate the effect of carrier phase multipath and to show increased availability under signal masking conditions.

INTRODUCTION

It is well known that GPS can be used to determine attitude parameters [e.g., Cohen 1992, Lu 1995]. Fixing the double difference carrier phase integer ambiguities is a necessity in order to achieve a good level of accuracy. The double difference (DD) technique differences the carrier phase observations twice, namely between antennas and between satellites, to cancel the errors due to satellite and receiver clock offsets. The DD technique also reduces the errors due to orbits and the atmosphere. For short baselines (up to tens of meters), these errors are negligible. However, one of the requirements for doing double differencing is to have at least four satellites commonly seen by the two antennas. In addition, the availability of extra satellites is highly desirable as it provides redundancy, accelerates the integer ambiguity search process, and increases the success rate. This strict requirement may pose significant difficulties in areas of reduced visibility, such as vehicular navigation in urban canyons or under foliage, marine navigation in fjords, etc. There have been several solutions proposed to solve the problem of signal shading. One that has been widely used in practice is integrating GPS with INS (inertial navigation system) measurements to determine the attitude parameters. The cost of an INS, however, makes the integrated system not appropriate for many applications. One of the alternatives is to augment GPS with GLONASS, its Russian counterpart. The improvements in availability, geometry, and reliability are direct advantages of integrating GPS and GLONASS.

Preliminary results for heading and pitch estimation using combined GPS/GLONASS have been presented by Keong and Lachapelle [1998]. The augmentation is not straightforward because GLONASS employs the technique of frequency division multiple access (FDMA) to distinguish the signals from different satellites, rather than the code division multiple access (CDMA) technique used by GPS. FDMA allows each GLONASS signal
broadcasts in its own different frequency and this makes the DD of carrier phase observables no longer possible without modification. The use of between-receiver carrier phase single differencing (SD) was proposed to get around with the problem. In this case however, receiver clocks and other errors do not cancel out. An external clock can be used as the common oscillator for the two receivers in order to reduce these errors. Remaining time and other biases are estimated using a low pass filter. The estimated biases can then be fed back to the carrier phase observations. A slightly modified On-the-fly (OTF) technique is then used for single integer ambiguity resolution. The ambiguity resolution technique can be modified to incorporate a known baseline constraint and/or external sensor information.

A brief review of GPS attitude determination and a brief overview of GLONASS are presented in the next two sections. It is then followed by a detailed discussion of problems encountered in GLONASS augmentation, with emphasis on the effect of multipath and the effect of clock errors and line biases. Static and kinematic tests are carried out to evaluate the proposed approach using two Ashtech GPS/GLONASS GG24™ receivers.

**REVIEW OF GPS ATTITUDE DETERMINATION**

The theory of attitude determination using multiple GPS antennas is well documented in the literature [e.g. Cohen 1992, Lu 1995] and will not be repeated here. If the relative position of two antennas can be determined with a sub-centimeter accuracy using the carrier phase observables of at least two points in space, two of the three attitude parameters, usually heading (ψ) and pitch (θ), of the platform can be estimated. As shown in Figure 1, heading is the rotation angle about the z-axis (up), clockwise being positive. Pitch is the rotation angle about the rotated x-axis (east), upward being positive:

\[
\begin{align*}
\psi &= \arctan \frac{\Delta E}{\Delta N} \quad \text{(rad)} \\
\theta &= \arctan \frac{\Delta U_p}{\sqrt{\Delta E^2 + \Delta N^2}} \quad \text{(rad)}
\end{align*}
\]  

(1)

In a twin-receiver system, the origin is at Antenna 1. Both the heading and the pitch are defined by the baseline from antenna 1 to antenna 2. In some instances, the baseline formed by two antennas may be parallel to the true heading direction of the vehicle. If this is the case, the GPS determined heading is equal to the heading of the platform. In most cases however, it is very difficult to set up the GPS antennas exactly parallel to the vehicle’s heading direction. Therefore, a misalignment angle between the two headings needs to be determined and taken into account [e.g., Sun et al 1997].

**REVIEW OF GLONASS**

Detailed discussions of GLONASS can be found in the literature, e.g. Daly & Misra [1996], Coordinational Scientific Information Centre [1995], and Langley [1997]. Since mid-1997, the number of healthy GLONASS space vehicles (SV) has been reduced to between 13 and 17. Therefore, the Dilution of Precision (DOP) of GLONASS is higher than that of GPS. On the other hand, the DOP of GPS/GLONASS is significantly better than either stand-alone system [e.g., Hall et al 1997]. This suggests that GLONASS can be a valuable tool in augmenting GPS.

There are many direct advantages in augmenting GPS with GLONASS for attitude determination. First and foremost, it is obvious that there are more satellites with the combined system. As a result, the satellite availability problem is reduced, especially under signal masking conditions. The geometry of the satellites, characterized by various DOPs, is likely to be strengthened. GLONASS, as an autonomous system completely independent of GPS, also offers additional reliability to the attitude estimates. Given the cost of combined GPS/GLONASS receivers, augmentation is relatively cost effective compared to other methods.

When combining GPS and GLONASS however, some of the major differences between the two systems have to be accounted for. These differences include the different time reference frames, coordinate reference systems, and the signal modulation techniques. GLONASS uses the Universal Coordinate Time, Soviet Union standard (UTC-SU) as its reference time frame. The Russian National Time and Frequency Services maintain UTC-SU. UTC-SU is kept within the international standard of UTC, UTC-BIPM (Bureau International des Poids et Mesures) within 1 microsecond (ms), since Russia made an adjustment of 9 ms in late 1996. GLONASS Time (GLONASST) is the time standard of the system. It has a known offset of three hours from UTC-SU due to the geographic location of Moscow. The difference between GLONASST and UTC-SU is known to 50 nanoseconds (ns). GPS, on the other hand, is referenced to UTC.
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transform the coordinates from pz90 to wgs84.
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baselines, typically 1 to 10 meters, the accuracy of many
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transformation parameters between wgs84 and pz90
based on the laser data using the orbital method. The two
transformations are both fully populated, i.e. all seven x-,
y-, z-axes translations and rotations and the scale in the
3-D similarity transformation are found to be significant.

Glonass FDMA

The major problem in integrating glonass and GPS arises
from the fact that glonass uses the fdma technique to distinguish the signals from different
satellites. GLONASS FDMA uses common PRN codes but different carrier frequencies. The frequency offset
between satellites in the L4 band is 562.5 kHz and that in
L2 is 437.5 kHz:
f_1 = 1602 MHz + k*0.5625 MHz
f_2 = 1246 MHz + k*0.4375 MHz
(2)

where k is the channel number ranges from 0 to 24.

The FDMA nature of GLONASS gives rise to a few
problems when attempting to use the standard double
difference technique. Firstly, the integer characteristic of the ambiguity term after between-satellite differencing
can not be preserved. Secondly, each of the glonass
signals suffers a different delay. Thus, the delay cannot
be estimated as part of the clock term as in the case of
GPS. Moreover, these delays are temperature dependent.
Thirdly, the fact that each measurements are made at
slightly different times in the two receivers due to clock
errors means that an extra error is present when the clock
errors are scaled by the different frequencies of the
GLONASS satellites. Again, this is not a problem for
GPS since there is only one frequency. And finally, a
much wider bandwidth is used in the channel design to
receive the range of GLONASS frequencies. This potentially leads to noisier observations and may have a
negative impact on ambiguity resolution. To get around
this problem, the standard DD mechanism has to be
modified to accommodate the different satellite
frequencies.

Modified DD for Glonass

Several solutions have been discussed to yield DD
solutions for glonass [e.g., Leick 1998]. The first
solution proposed is to estimate the receiver clock term
from the pseudoranges and substitute it back in the DD
carrier phase equations accordingly. To achieve this, at
least four satellites must be in view and their
pseudoranges must be measured with high accuracy.
Current technology allows the receiver clock term to be
estimated from pseudoranges to an accuracy of only
several ns. However, an error in the receiver clock term
of 1 ns causes a phase error in double difference of about 30 cm. This corresponds to about 0.01 cycles (given that 2 GLONASS frequencies are separated by, for example, 11 MHz, which is about 27 meters in wavelength). Therefore, estimating the receiver clock term from pseudoranges is not satisfactory with the current technology [Leick 1998].

The second solution is to scale the L1 GLONASS frequency band to a mean frequency. With this, the receiver clock term can also be eliminated when all the GLONASS carrier phase observables are scaled to a mean frequency. The mean frequency, for example, can be located in the middle of the L1 GLONASS frequency band of 1.60875 GHz. The drawback with this solution is that the new double difference ambiguity term is no longer an integer, as in the case of GPS. Most of the ambiguity resolution techniques cannot be readily used with this approach. Not having the ambiguity fixed results in having accuracy in the decimeter range [e.g., Landau & Vollath 1996]. Clearly, this level of accuracy is insufficient for attitude determination.

The third solution is to scale the entire L1 GLONASS frequency band to a common frequency. This preserves the integer nature of the ambiguity term after double differencing by finding an auxiliary wavelength. However, as shown by Rossbach & Hein [1996], the auxiliary wavelength is extremely small for the GLONASS L1 carrier. The 65 μm auxiliary wavelength makes integer ambiguity resolution impossible due to noise and geometry.

GLONASS double differencing could be used if one has access to dual frequency carrier phase observables [e.g., Leick et al 1995]. This approach is similar to the resolution of the widelane ambiguity using dual-frequency GPS measurements. The limitation to this approach is the need for a dual-frequency receiver and the need for accurate pseudoranges to compute the initial ambiguity estimate with sufficient confidence. The noise of widelane carrier phase observables is relatively higher and this would result in an accuracy deterioration of the attitude parameters.

The above techniques are complicated and may not be applicable to attitude determinations. Alternatively, one can use the technique of SD with a common oscillator. The SD approach is used in this study and is described below.

**SD WITH A COMMON CLOCK**

The carrier phase observation equation for GPS can be written as:

\[ \Phi = \rho + d\rho + \lambda N + cdt - cdt - \Delta lb + dTrop + dlb + dIon + \epsilon \]  \hspace{1cm} (3)

where,
- \( \Phi \) is the measured carrier phase in unit of length
- \( \rho \) is the true (but unknown) range
- \( d\rho \) is the orbital error
- \( N \) is the integer ambiguity
- \( cdt \) is the satellite clock offset (from GPST)
- \( cdT \) is the receiver clock offset (from GPST)
- \( lb \) is the line bias delay caused by the physical length of the cable
- \( dTrop \) is the tropospheric delay
- \( dlb \) is the ionospheric delay; and
- \( \epsilon \) is the error term which includes measurement noise, multipath and etc.

Differencing the above equation between two receivers results in:

\[ \Delta \Phi = \Delta \rho + \Delta d\rho + \lambda \Delta N - c \Delta dT - \Delta lb + \Delta dTrop - \Delta dlb + \Delta dIon + \Delta \epsilon \]  \hspace{1cm} (4)

where \( \Delta \) is the SD operator for differencing between the two receivers. The orbital and atmospheric errors in Equation (4) practically cancel out for a baseline length of a few meters. The satellite clock error, \( cdt \), is completely eliminated. However, SD does not eliminate the receiver clock error, \( cdT \), and the line bias, \( clb \), as in the case of DD. Nor does it eliminate the integer ambiguity term \( N \) as in the case of the between-epoch single difference. The random errors \( \epsilon \) includes both multipath and receiver noise and they are amplified by a factor of \( v^2 \). The amplification is smaller than that of DD, which doubles the magnitude of noise.

By re-arranging the terms and neglecting the errors that cancel out, Equation (4) can be re-written as:

\[ \lambda \Delta N = \Delta \Phi - \Delta \rho + c \Delta dT - \Delta lb + \Delta \epsilon \]  \hspace{1cm} (5)

where the \( \Delta \Phi \) term is the SD GPS receiver measurement the \( \Delta \rho \) term is calculated and the \( c \Delta dT \) and \( \Delta lb \) terms are modeled.

If a common oscillator is used to generate time and frequency for the two receiver clocks, the \( cdT \) term of each receiver will have the same drift. However, this does not imply that the receiver clock offset for each of the receiver is identical. The reason for this is that the clock only provides the two GPS receivers with a stream of pulses with no absolute time tag. Thus, there is an initial bias for each receiver and the two initial biases are likely to be different. The other way to see this problem is to decompose the \( cdT \) term in Equation (3) into two parts. This first part is an absolute bias from a perfect clock and the second part is its drift over time from a perfect clock. Driving two receivers with a common oscillator only ensure that the drift part of \( cdT \) is identical, but there is no guarantee that the first part of \( cdT \) is the same for both receivers. Therefore, it should be intuitive to conclude that the \( c \Delta dT \) in Equation (5) is indeed a non-zero constant term.
The line bias term $\Delta lb$ is due to the different cable lengths to the two antennas. Therefore, it is a constant term that does not cancel out after SD. The magnitude of this $\Delta lb$ term is equal (or somewhat equal due to the different propagation speed of the cable and different cable materials) to the difference in cable length between the two receivers.

Given the fact that both the effects of the clock error and the line bias are constant, the combined effect is of a random bias nature. A random bias has the characteristic that it can be of any value prior to determination. However, after system initialization, the value does not drift with time. This combined effect can easily reach several tens of meters. However, the total effect will not show up in the residuals. The fact that the combined effect is constant makes multiples of the $c\Delta T$ and $\Delta lb$ terms absorbed by the $\Delta N$ term when solving for $\Delta N$, as shown in Equation (5). Therefore, only the remaining fractional part of the combined $c\Delta T$ and $\Delta lb$ terms will show up as a constant bias in the single difference residuals. In fact, any unmodeled error between the two receivers which is constant over time will combine with the above two effects to have their fractional part show up in the residuals. The theoretical maximum of the combined effect should therefore be no larger than $1/2 \lambda$, which is 9.51 cm for $L_1$ (GPS). Also, this residual bias should be similar for all SVs. This is shown in Figure 2 where there is a $-0.15$ cycle bias for the four SVs.

The residual bias is estimated using a simple averaging filter using equation, namely:

$$\text{est}_i = \frac{i - 1}{i} \text{est}_{i-1} + \frac{1}{i} \text{res}_i,$$

where “est” is the estimate of the residual bias and “res” is the residual bias. The estimates of the residual biases for the four satellites shown are found to agree among each other within 0.03 cycle after the averaging filter converges to a steady state. This can be seen in Figure 3. The “fractional parts” of the residuals, which have a magnitude typically at the centimeter level, can be disastrous for integer ambiguity fixing. They must be estimated and fed back into the carrier phase observables for removal. Once the biases are estimated and fed back to the carrier phase observables, a zero-mean should be obtained. This can be seen in Figure 4. The remaining 1-cm noise is due to multipath, which does not cancel out.

During the convergence phase of the filter, the estimated attitude parameter accuracy will be lower than once the filter has converged. In addition, the residual bias estimates are affected by multipath. Estimate errors will increase the SD carrier phase observable noise and this will further degrade estimated attitude parameter accuracy. The latter is therefore expected to be somewhat lower than the corresponding accuracy derived using a DD approach in the case of GPS.

![Figure 2: SD Residuals for Selected GPS SVs](image2)

![Figure 3: SD Residual Bias Estimates for Four GPS SVs](image3)

![Figure 4: SD GPS Residuals after Bias Estimate Feedback](image4)
experiment was carried out in order to verify if the residuals are indeed constant over time even when no satellite is tracked. A static test was conducted for this purpose. The antenna baseline vector was 12 meters and a 2-hour data set was collected. Both antennas were shaded at the same time for about 10 minutes during the test so that no satellite could be tracked during the shading period. The equipment consisted of two Ashtech 24-channel GG24™ receivers. A splitter was used to split the signal from the common oscillator, a 10 MHz output rubidium clock, to the two receivers. The software implementation of the SD GPS/GLONASS approach was realized by modifying the University of Calgary HEAD™ software [Sun et al 1997]. The estimated heading and pitch were 89.80° and -0.46°, respectively. The heading and pitch estimates with the 2-sigma bounds as obtained by the filter are plotted in Figure 5 and 6.

The residuals with the PRN31 bias is shown in the upper part of Figure 7. It is obvious that there is a +4.5 cm (+0.23 of a L1 cycle) bias in this case. Other satellites (not shown) also contain biases of +4 to +5 cm. Using Equation (6), the estimated residual bias for PRN31 is found to be +4.53 cm and is plotted in the middle part Figure 7 with an offset –1.5 cm for clarity. This estimated bias is then fed back to the carrier phase measurements and a zero-mean is obtained. The residuals after bias removed are then plotted in the lower part of Figure 7. This 1-2 cm noise is typical of multipath. The results of this test show that the effects of clock errors and line bias terms in Equation (5) are indeed constant.

MULTIPATH

As can be seen in Figure 7, the effect of multipath typically reaches 1-2 cm. This level of relative positioning accuracy translates to an accuracy of ℓ in attitude determination when a 1-meter inter-antenna separation is used. Carrier phase multipath remains the most important error source for attitude determination. A more detailed discussion of carrier phase multipath can be found in [Braasch 1996, Georgiadou & Kleusberg 1988].

In order to illustrate the effect of GPS and GLONASS carrier phase multipath on platform attitude estimation, static tests with the twin GPS/GLONASS receiver system described earlier were performed on the lower roof of the Engineering Building of the University of Calgary, as shown in Figure 8. The multipath environment is moderate. The inter-antenna separation was 12 meters. Since the antennas are stationary, the estimated heading and pitch of the platform should be constant from epoch to epoch. Time series analysis thus provides a way to analyze multipath effects on the estimated heading and pitch components. It should be noted that, compared to multipath, receiver noise only causes small random changes in the estimated attitude parameters while multipath repeat itself.
investigate the effect of multipath at a site using data observed over two consecutive days, with the second day moved back 4 minutes in time. On the other hand, GLONASS satellites have the same nominal orbital period of 675.7 minutes, but their orbits produce ground tracks that repeat every 17 orbits or every 8 solar days less 32 minutes [e.g., Daly & Misra 1996]. Therefore, the GPS/GLONASS satellite-reflector-receiver geometry is similar after about eight days. In a fixed static environment, multipath errors from combined GPS and GLONASS will repeat themselves after about eight days.

Shown in Figures 9 and 10 are the heading and pitch components estimated using GPS/GLONASS measurements from epoch to epoch for Day 1 and Day 9. The reference heading is 89.79° and the reference pitch is −0.05°. There was no chokering ground plane used in the experiment and there was no elevation mask angle limit used. It can be seen, especially from Figure 10, that the variation pattern of the estimated heading and pitch parameters for the eighth day is quite similar to that of the first day. The results from the eighth day are shifted upward by 0.2° for clarity. Centered at 89.5° in Figure 9 and 0.4° in Figure 10 are the heading and pitch differences between the two days. The long-term variations due to multipath have been eliminated. The statistics of the above results can be found in Table 1.

| Table 1: Statistics for Heading and Pitch Estimates for Day 1 and Day 9 – Static Test |
|-----------------------------------|-----------|-----------|----------|
| Heading                           | Day 1     | Day 9     | Diff.    |
| Mean                              | -89.789°  | -89.798°  | 0.009°   |
| Std. Dev.                         | 0.011°    | 0.011°    | 0.0009°  |
| Pitch                             | Day 1     | Day 9     | Diff.    |
| Mean                              | -0.051°   | -0.054°   | -0.003°  |
| Std. Dev.                         | 0.029°    | 0.027°    | 0.002°   |

The computed a posteriori standard deviation in this test is 0.01° for heading and 0.03° for pitch. The relative baseline positioning accuracy, translated from the above computed standard deviation for attitude parameters, is about 2 mm to 6 mm by using an inter-antenna separation of 12 meters. This level of accuracy is within the expected accuracy range of carrier phase positioning in a moderate multipath environment. The computed a posteriori standard deviation for the residuals is about 0.01° for both heading and pitch and it translates to about 2 mm. Since the residuals has been differenced (between Day 1 and Day 9) once, the relative baseline positioning accuracy is about 1.4 mm. This level of accuracy is also within the expected accuracy range of carrier phase noise with low multipath.

The jump in the pitch component at epoch 218 is caused by the change in satellite constellation. The loss of a low elevation satellite (SV15) caused the noticeable jump in the pitch component but very little in the heading component. Depending on the relative geometry between the satellites and baseline vector, the effect of losing or gaining a satellite on heading and pitch estimates may be different. Usually, the low elevation satellites significantly strengthen the satellite geometry but these satellites are much more affected by multipath [Lu, 1995]. In order to achieve the highest possible accuracy in attitude determination, multipath should be reduced, if not avoided, as much as possible.

If the environment and equipment can be controlled, methods used to reduce the effect of multipath are to have a careful site selection, to use ground planes such as chokering [Lu et al 1993], and/or to use on-line receiver multipath reduction techniques [e.g., van Nee et al 1994]. If the above conditions are beyond the control of the users, which is more likely the case in attitude determination, mathematical modeling of multipath signatures is indeed required. Efforts were made by several research groups to study and mitigate the errors generated from carrier phase multipath. Georgiadou & Kleusberg [1988] used L4-L5 measurements to estimate the carrier phase multipath error and the carrier wavelength. Axalrad et al [1994] have exploited the signal-to-noise (SNR) information from the receiver in post mission, along with the antenna gain patterns, to estimate multipath. Ray et al [1998] more recently investigated the
effect of carrier phase multipath and developed a system to reduce the effect using multiple closely-spaced antennas. The technique estimated the parameters of the composite multipath signal in a Kalman filter and removed the error due to all multipath signals in static mode.

GPS/GLONASS KINEMATIC TEST

A kinematic test was conducted in an unobstructed open sky area to validate the SD approach in such an environment. The data was collected over a period of 1.5 hours on February 20, 1998. The system was installed on the roof of a vehicle. The equipment consisted of two GG24™ receivers described earlier. A splitter was used to split the power from the common 10 MHz output rubidium oscillator. The length of the baseline was 1.1 meter. The vehicle speed ranged from 20 km/h to 80 km/h. The pitch angle variation was ±8°. Reference heading and pitch estimates, shown in Figure 11, were obtained using the GPS measurements only.

A 10° elevation mask angle was used. The number of GPS satellites remained at seven to eight, whereas the number of GLONASS satellites stayed at four throughout the period. The ADOP (Azimuth or heading DOP) of the combined solution is not much better than that of GPS and both were = 1.0 most of the time. GLONASS augmentation however did improve the EDOP (Elevation or pitch DOP). During the interval between 244300 s and 246050 s, the combined GPS/GLONASS reduced the EDOP to 1.2, from 1.8 for GPS alone, as shown in Figure 12. The heading and pitch estimates with GPS and GPS/GLONASS can be seen in Figure 13. The GPS/GLONASS estimates are offset by 5° for heading and pitch for clarity. The statistics are summarized in Table 2. The unaided GPS solution is obtained with the SD method while the reference solution is obtained with the DD method as described earlier. The agreement between the two solutions provides a measure of the compatibility of the two approaches since they are both based on the same measurements.

GLONASS augmentation increases availability by about 3%. In the gaps where no heading and pitch were estimated due to ambiguity fixing failure, GLONASS augmentation resulted in faster recovery than GPS alone. A blunder is defined as an estimated value being different from the reference value by more than three sigmas and is mainly caused by a wrong ambiguity set being selected by the OTF algorithm. Thus, the number of blunders may be used as a reliability measure. The percentage of blunders is about the same with and without GLONASS augmentation. The GPS and GG mean and RMS differences for both heading and pitch estimates were statistically similar. This is unexpected given the additional error sources such as line biases and temperature effects on frequency differences [e.g. Dodson et al 1998].
Another kinematic test in a less open space residential area was conducted to investigate the system performance under reduced visibility. The number of GPS satellites varied from four to seven, whereas the number of GLONASS satellites stayed mostly at two throughout the period. The ADOP and EDOP of the combined solution is about 0.5 better than the corresponding values for GPS alone. Figure 14 shows the number of satellites and the associated ADOP and EDOP for this case. The heading and pitch estimates with unaided GPS and GPS/GLONASS can be seen in Figure 15. The GPS/GLONASS estimates are offset by 5° for heading and pitch for clarity. The statistics are also summarized in Table 2. Availability improved by 8% when GLONASS augmentation is used. However, the percentage of blunders also increased, namely by 2.9%. The cause of this is still being investigated and is likely due to either or both a filter tuning problem or the additional noise in the GLONASS measurements.

### Table 2: Statistics of GPS and GLONASS Augmentation for Open Areas and Residential Areas

<table>
<thead>
<tr>
<th></th>
<th>Mean/ RMS $\Psi$</th>
<th>Mean/ RMS $\theta$</th>
<th>Availability</th>
<th>Blunder</th>
</tr>
</thead>
<tbody>
<tr>
<td>GPS open sky</td>
<td>0.02$^o$</td>
<td>-0.06$^o$</td>
<td>89%</td>
<td>1.8%</td>
</tr>
<tr>
<td>GG open sky</td>
<td>0.15$^o$</td>
<td>-0.16$^o$</td>
<td>91%</td>
<td>2.2%</td>
</tr>
<tr>
<td>GPS residential</td>
<td>0.17$^o$</td>
<td>0.20$^o$</td>
<td>71%</td>
<td>7.2%</td>
</tr>
<tr>
<td>GG residential</td>
<td>0.04$^o$</td>
<td>-0.25$^o$</td>
<td>79%</td>
<td>10.1%</td>
</tr>
</tbody>
</table>

### CONCLUSIONS

The use of between-receiver single differencing with a common oscillator has been demonstrated for both GPS and combined GPS/GLONASS. Even with a GLONASS constellation of only 14 satellites, GLONASS augmentation improves the availability of attitude determination system. This is especially true under signal masking conditions. The ADOP and EDOP of GPS/GLONASS are better than those of GPS alone are. However, the improvement in availability comes at the cost of a lower heading and pitch accuracy. This is due to the limits of the single difference approach and the FDMA technique used by GLONASS. Residual clock and line biases have to be estimated with a filter and contribute to increasing the single difference observable noise. This could also be the reason for a lack of reliability improvement for this application. Several effects are still being investigated by the author, namely the use of a lower cost common frequency and time standard, the effect of inter-receiver inter-channel biases, and the effect of temperature changes on the stability of the solution.

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### REFERENCES


