New Decision Variables for GNSS Acquisition in the Presence of CW Interference

Mohammad Abdizadeh, James T. Curran, Gérard Lachapelle

Index Terms—Acquisition, Averaging, CW, Decision metrics, Detection, Differencing, GNSS, GPS, Interference, Jamming.

Abstract—This paper investigates the problem of GNSS signal acquisition using a consumer-grade receiver in the presence of CW interference. By examining in detail the impact of interference on the acquisition search space, a set of decision variables are developed. These decision variables leverage the systematic properties of the interference observed in the search space; manipulating or differencing portions of it to achieve improved detection performance. Strategies based on these variables are examined under a selection of operating scenarios including acquisition in the presence of interference when the receiver has some, or has no a priori information regarding the interference. It is shown that a receiver employing these new schemes can tolerate up to 10 dB more interference power than a traditional receiver.

I. INTRODUCTION

In recent years, in order to combat the ever increasing threat of Radio Frequency (RF) interference, the study of its effect on Global Navigation Satellite System (GNSS) receivers has attracted significant attention [1]-[10]. The exact impact of interference on the GNSS receiver performance is heavily dependent on the receiver configuration but, in general, interference induces losses in the quantization process, impairs symbol recovery [11], and can significantly degrade acquisition and tracking loop performance [11].

Amongst the various types of interference signals that can be observed globally, Continuous Wave (CW) interference has emerged as the most significant threat to GNSS receivers [1] [12] [13]. Over the last number of years, this threat has grown rapidly due to the increased prevalence of commercially available GNSS jammers or Personal-Privacy Devices (PPD) [12] [14]. Unfortunately, emanations from such PPDs are often far-reaching and can cause large-scale disturbances to GNSS devices. The consumer-grade receiver, which is possibly the most widely used receiver type, is also likely the most vulnerable to such interference. Such receivers are typically equipped with poor quality components and low resolution digitizers and, as a result, tend to exhibit a relatively small dynamic range. Thus, these receivers can be sensitive to Automatic Gain Control (AGC) calibration and very susceptible to jamming.

The presence of interference within received signal increases the challenge of acquisition of the desired GNSS signal. Previous work has mainly considered the effect of interference on the conventional acquisition schemes of the GNSS receiver [3] [4]. For example in [3], the effects of narrowband CW interference on the traditional acquisition process have been extensively examined wherein a statistical model is presented which focuses on cell-level statistics. Theoretical results of both the false-alarm and the detection probabilities of a GNSS receiver are given in the presence of CW interference. In [6], the effect of Amplitude Modulation (AM) and Frequency Modulation (FM) interference on the acquisition process of a receiver employing different values of coherent and non-coherent integration time is presented based on a software receiver implementation. It was shown that AM interference introduces more distortion than FM interference and the results of various interference powers with different coherent integration time were presented. Other current approaches include methods based on the adaptive array antennas, for example [9] [10], although these techniques are deemed to be beyond the scope of this work.

To the best of the author’s knowledge, the acquisition of quantized signals in the presence of interference is not considered in the literature. Receiver non-idealities such as sampling and quantization, which become pronounced when strong interfering signals are present, have yet to be considered. In addition, it is shown here that knowledge of the presence of interference in the received signal, can not only reduce the likelihood of false acquisition, but can also increase the probability of detecting the correct signal if, as will be illustrated here, the decision variable is appropriately modified. In terms of acquisition performance, the challenges introduced by interference and the relative merits of various decision variables can be assessed at the cell-level or at the system-level.

Within this work, the cell-level performance relates to the effective probability of false-alarm when the signal is
absent and the probability of detection when the signal is present [15, 16]. Another useful and related metric, known as the system-level performance is defined in terms of the overall probability of correct detection of the signal, considering the entire search space [15, 16]. In this case the detection probability is a function of: the cell-level probability of detection, for the cell which contains the signal; the cell-level false-alarm probability for all other cells in the search space; and on the particular sequence in which the cells are tested, the simplest case of which, namely the parallel search, is employed in this work [15, 16].

This paper focuses on consumer-grade GNSS receivers in light of their popularity and their particular vulnerability to jamming. Modern consumer-grade receivers are mostly equipped with 1-, 2- or 3-bit quantizers [17], and employ a low sampling rate. The Receiver Operating Characteristic (ROC) performance of the acquisition process is degraded due to distortion caused by employing such quantizers. Interference increases this degradation, resulting in reduced Carrier-to-Noise-Floor-Ratio ($C/N_0$). Here, the effect of the quantization process on the GNSS signal acquisition will be considered, paying particular attention to the cell-level detection performance. The results of the probability of detection and false-alarm, determining the ROC, for the more popular 1-, 2-, 3-bit quantizers will be computed to identify and quantify the benefits of increasing quantizer resolution.

An analysis is presented of the impact of fixed-frequency and fixed-amplitude CW interference and its impact on the performance of traditional acquisition algorithms. The presence of interference not only decreases the acquisition performance at the cell-level, in the sense of ROC performance, but also causes system-level performance degradation. The traditional parallel search scheme (see, for example, [1, 15, 16]) is chosen as a benchmark against which to measure the relative performance of modified and improved search strategies.

Traditional acquisition techniques have been conceived and developed to operate in the presence of thermal noise alone; thermal noise and multiple-access interference; and, in some cases, thermal noise and a multi-path channel [18, 19]. These techniques have almost invariably been based around a traditional power detector. When considering the problem of jamming and interference, much of the previous work has assessed its impact on this detector, or some slight variation thereof. In contrast, this work aims to develop and evaluate the performance of a novel set of acquisition detectors which have been specifically designed to operate in the presence of strong interfering signals.

Three acquisition cases will be considered, differing in terms of the receiver’s $a$ priori knowledge of the interference presence and characteristics. In the first case, the receiver has no information about the interference presence and blindly performs some acquisition algorithm to acquire the GNSS signal. In the second case, the receiver only knows that the interference is present in the received signal. Here, it is assumed that this information is provided to the acquisition process by some interference detection methods [20]. The third case assumes that the receiver not only is aware of the interference presence, but it also has an estimate of one, or other, or both of the interference instantaneous frequency and amplitude.

For each of these three cases, a suitable detector function is defined which leverages knowledge of the properties of an acquisition search and any $a$ priori knowledge of the interference presence and characteristics. The characteristic features of the acquisition search space in the presence of strong interference are identified and are used to construct three novel detector functions: a window-based function; a cell-pairing based function; and a non-coherent interference removal function. These detectors are then analyzed and their performance, relative to the traditional detector, is examined under a range of operating conditions.

This paper is organized as follows: the receiver model, signal structure, and acquisition search space is described in Section II. Section III elaborates on the characteristics of the signal search space, considering the effects of interference and front-end quantization. The cell-level ROC performances of both the traditional and a selection of new, improved, acquisition decision statistics are presented in Section IV. Section V extends these results to the system-level and examines the benefits of using $a$ priori information regarding the interference, in the acquisition process. These results are supported by extensive Monte-Carlo computer simulation.

II. SIGNAL AND RECEIVER MODEL

Fig. 1 shows a simplified block diagram of a typical GNSS receiver, wherein it is assumed that the received signal at the antenna is a single satellite signal distorted by a zero mean Additive White Gaussian Noise (AWGN) and a narrowband interference. After radio frequency processing which may include multiple down-conversions and filtering stages, the Intermediate Frequency (IF) signal can be modelled as:

\[ r_{\text{IF}}(t) = s_{\text{IF}}(t) + i_{\text{IF}}(t) + \eta_{\text{IF}}(t). \]  

(1)
This signal is comprised of a GNSS signal, $s_{\text{IF}}(t)$, a narrowband interference, $i_{\text{IF}}(t)$, and an additive noise, $\eta_{\text{IF}}(t)$. The GNSS signal component is defined as [21]:

$$s_{\text{IF}}(t) = A_c c(t - \tau) d(t - \tau) \times \cos(2\pi(f_{\text{IF}} + f_D)t + \theta), \quad (2)$$

where, $A_c$ is the amplitude of the received signal after the IF stage, $c(t)$ is the spreading code with chip period $T_c$, $d(t)$ represents the data modulation, $\tau$ is the initial code phase, $f_{\text{IF}}$ is the IF frequency, $f_D$ is the Doppler shift, and $\theta$ is the initial IF carrier phase. The noise component is assumed to be a zero mean AWGN with a single-sided power spectral density of $N_0$ W/Hz. The interference signal is modeled as a simple tone as described in Section II-B.

### A. Quantization

Typically, a low resolution quantizer is utilized in consumer-grade GNSS receivers [17, 22]. The reasons for such a choice include the facts that employing a low resolution quantizer is efficient in the sense of cost, computational requirement, and data-throughput. Furthermore, with an appropriate sampling rate, the quantizer degradation, for a purely AWGN channel, is not significant, being 1.96 dB, 0.55 dB, and 0.16 dB for 1-, 2-, and 3-bit quantizers, respectively [17, 21]. While this degradation is quite benign, it is intensified in the presence of interference [11]. Hence, the configuration of the quantizer can have a significant effect on the performance of interference-suffering GNSS receiver. In order to provide reliable receiver performance under strong jamming/interference conditions, the design of the AGC system and the quantizer should be carefully considered.

Previous work [11, 17, 23] has shown that there is an optimum AGC for multi-bit quantizers which results in minimum Bit Error Rate (BER) or, equivalently, maximum effective carrier to noise ratio. It is expected that a receiver can adaptively adjust the quantizer gain both in the presence and absence of interference to minimize the total processing losses. This will provide some associated improvements in acquisition and tracking performance. This gain is a function of noise and interference power with respect to desired signal power. In order to set the AGC gain optimally, not only should the noise level be estimated, but also the interference power must also be determined [11].

### B. Continuous Wave Interference

The interference component, $i_{\text{IF}}(t)$, of (1) is assumed to be a fixed CW interference modeled here as a pure sinusoidal tone having frequency offset of $f_\Delta$ relative to the nominal IF frequency, such that $f_{\text{int}} = f_{\text{IF}} + f_\Delta$. The transmitted interference amplitude, $A_{\text{int}}$, is assumed constant. The fixed CW interference signal can, therefore, be represented by:

$$i_{\text{IF}}(t) = A_{\text{int}} \cos(2\pi f_{\text{int}} t + \theta_{\text{int}}) \quad (3)$$

where, $\theta_{\text{int}}$ is a random initial phase uniformly distributed on the interval $(-\pi, \pi]$.

Fig. 2 shows the normalized Power Spectral Density (PSD) of a CW jammer assuming a sample rate of 8 MHz where the both the IF and interference frequencies are equal to 0.5 MHz. This figure illustrates the spectrum for both a non-quantizing receiver (infinite resolution) as well as that for 1-, 2-, and 3-bit quantizers. Here, the value of the AGC gain has been optimized for GNSS signal processing [11]. As can be seen, quantization introduces several harmonics in the spectrum. The frequency and amplitude of these harmonics depend on the interference frequency, the sampling frequency and the particular quantizer configuration [24]. Under such conditions, low resolution quantizers, or those that are poorly configured, will induce more harmonics with higher relative powers [11].

As the quantization process distorts and scales the received signal, the performance of subsequent receiver operations are often related to the characteristics of the signal immediately prior to quantization, specifically, $r_{\text{IF}}(t)$. Similar to the common signal-to-noise-ratio metric, a JNR can be defined. Assuming that the sampling rate, $F_s = 1/T_s$, of the IF signal is equal to twice that of the IF front-end bandwidth, $B_{\text{IF}}$, the power of $\eta_{\text{IF}}(t)$, can be given by $\sigma^2_{\text{IF}} = B_{\text{IF}} N_0$. Moreover, the JNR can be defined as:

$$\text{JNR} = \frac{1}{2} \frac{A^2_{\text{int}}}{\sigma^2_{\text{IF}}} = \frac{A^2_{\text{int}}}{F_s N_0}. \quad (4)$$

### C. Signal Acquisition

To perform signal acquisition, the receiver must first demodulate the received signal using an estimate of the...
received signals code phase and carrier Doppler. In a conventional receiver, the received signal is multiplied by a delayed version of the local replica of the GNSS signal, comprising of a replica spreading code, with initial code phase $\hat{\tau}$, and a replica carrier with frequency $\hat{f}_D$ an arbitrary initial carrier phase. This product is then coherently integrated over a fixed period, denoted here by $T_I$, to produce a Cross Ambiguity Function (CAF), $S(\hat{\tau}, \hat{f}_D)$ (1). 

This complex valued function, in the absence of both noise and interference, will generally have maximum amplitude when the code phase and carrier frequencies of the received and local replica signals coincide. Therefore, the square magnitude of the value of the CAF is typically examined by an acquisition scheme. Although the CAF is a continuous function in $\hat{\tau}$ and $\hat{f}_D$, it is generally evaluated at discrete intervals where each $(\hat{\tau}, \hat{f}_D)$ pair is referred to as a cell. The CAF value for a given cell, $(\hat{\tau}, \hat{f}_D)$, can be rewritten as:

$$S(\hat{\tau}, \hat{f}_D) = S_y(\hat{\tau}, \hat{f}_D) + S_{\text{int}}(\hat{\tau}, \hat{f}_D) + S_{\eta}(\hat{\tau}, \hat{f}_D),$$

where, $S_y$, $S_{\text{int}}$ and $S_{\eta}$ are the contribution of the GNSS signal, the interference and noise, respectively.

As a mathematical convenience, this demodulation process is often represented by an equivalent filter with impulse response:

$$h_c[n] = \begin{cases} 
  c(nT_s - \hat{\tau}) e^{j2\pi(f_{\text{IF}} + \hat{f}_D)nT_s} & \forall 0 \leq n \leq \frac{T_I}{T_s} \\
  0 & \text{otherwise},
\end{cases}$$

which has a Fourier transform $H_c(f)$. From (6), the contribution to (5) from the GNSS signal can be represented by the convolution:

$$S_y(\hat{\tau}, \hat{f}_D) = \frac{A_s}{2} \text{sinc}(\pi \delta x T_I) R(\delta \hat{\tau}) e^{-j(\theta_0 + \pi \delta x T_I)},$$

(7)

where $\delta x$ represents the error in the receiver estimate, $\hat{x}$, of the parameter $x$, such that $\delta x = x - \hat{x}$ and $R(\delta \hat{\tau})$ represents the autocorrelation function of $c(t)$.

The interference component of $S(\hat{\tau}, \hat{f}_D)$, is given by [25]:

$$S_{\text{int}}(\hat{\tau}, \hat{f}_D) = h_c[n] * i_{\text{IF}}(nT_s)$$

$$= \frac{A_{\text{int}}}{2} \left\{ H_c(f_{\text{IF}} + f_{\text{int}} + \hat{f}_D) e^{j2\pi(f_{\text{IF}} + f_{\text{int}} + \hat{f}_D)\hat{\tau} + \theta_{\text{int}}} \\
+ H_c(f_{\text{IF}} - f_{\text{int}} + \hat{f}_D) e^{j2\pi(f_{\text{IF}} - f_{\text{int}} + \hat{f}_D)\hat{\tau} - \theta_{\text{int}}}) \right\},$$

(9)

which, for the non-quantizing receiver, has been thoroughly analyzed in [3]. For a quantizing receiver this interference will be distorted and will no longer resemble a pure sinusoid. By considering its Fourier series expansion, however, it may be represented as a linear combination of weighted sinusoids. In this way, (9) may be utilized to evaluate the contribution of each element in the series and $S_{\text{int}}(\hat{\tau}, \hat{f}_D)$ can be evaluated as their linear combination.

The contribution of the noise term, $\eta(t)$, to $S(\hat{\tau}, \hat{f}_D)$, is a zero mean circularly symmetric complex Gaussian random variable, and is given by [1]:

$$S_{\eta}(\hat{\tau}, \hat{f}_D) = \mathcal{N}(0, \frac{N_0}{2T_I} \mathcal{I}_2)$$

(10)

where, $\mathcal{I}_2$ is the $2 \times 2$ identity matrix and $\mathcal{N}(\mu, \sigma^2)$ represents a Gaussian random variable with mean $\mu$ and variance of $\sigma^2$. Although the noise contribution in adjacent cells in the search space will be correlated, this correlation is negligible in comparison to that of the interference and so, for simplicity, it is neglected here.

D. Real Data Collection and Monte-Carlo Simulation

In this paper, the acquisition of the GPS L1 C/A signal is considered as a case study. In order to generate a simulated interfered GNSS signal, Spirent GSS 7700 simulator controlled by the SimGEN software is employed [26]. The Spirent simulator is used to generate variety of interference powers and frequencies. A static antenna mode is considered here and the simulated noise floor is set to -130 dBm. The resulting signal are amplified by a Low Noise Amplifier (LNA) and subsequently down converted and digitized through a National Instrument RF front-end [27] resulting in 16-bit complex raw IF samples with $F_s = 5$ MHz. Data derived from this simulation technique was employed.
in Section III for the purposes of characterizing the acquisition search space.

In contrast, to generate the cell-level and system-level results in Sections IV and V, a more efficient simulation technique using MATLAB generated signals are employed. Assuming that the received signal has a fixed $C/N_0$ of 48 dB-Hz, Monte Carlo simulations, the configuration of which is summarized in Table I, are employed to evaluate the performance of various acquisition schemes. The frequency plan and noise figure are employed to evaluate the performance of various acquisition schemes. The frequency plan and noise figure are employed to evaluate the performance of various acquisition schemes. The frequency plan and noise figure are employed to evaluate the performance of various acquisition schemes. The frequency plan and noise figure are employed to evaluate the performance of various acquisition schemes. The frequency plan and noise figure are employed to evaluate the performance of various acquisition schemes.

In both of the above simulation techniques, a coherent integration period commensurate with the harsh environment under consideration is chosen, specifically $T_I = 7.0$ ms is used as it effects a suitable trade-off between correlation gain and data-modulation related losses \[28\]. To minimize losses due to residual Doppler and code delay error, the acquisition search space was configured to have a code bin width, $\Delta \tau$, of 0.5 chips and a Doppler bin width, $\Delta F$, of 100.0 Hz.

### III. Search Space Properties

In the presence of interference, the statistics of the CAF across the search space vary depending upon the interference characteristics. Fig. 3 shows a CAF for a received signal corrupted by CW interference, with JNR = -10 dB and $f_\Delta \approx 100$ Hz. For the interference free case, the CAF values exhibit a maximum value located at the correct code delay and carrier Doppler and, therefore, conventional methods based on comparing the CAF level to a threshold can be employed. However, as it can be seen for CAF values in the presence of any appreciable interference, the correct code delay and carrier Doppler no longer correspond to the CAF global maximum. As a result, traditional threshold comparing is no longer effective. In this section, the properties of the CAF in the presence of interference are examined.

Acquiring the correct cell inside search space is generally a pattern recognition problem. In the interference free case, however, this problem can be reduced to that of a simple detection or detection-and-verification scheme \[29\]. Generally, a decision variable is computed as some function of one or more conservative samples of the CAF. For example, a widely used decision variable, denoted here by $D$, considers the non-coherent combinations of the $K$ conservative samples:

$$D(\hat{\tau}, \hat{f}_D) = \sum_{k=0}^{K-1} \left| S_k(\hat{\tau}, \hat{f}_D) \right|^2,$$

where, the subscript, $k$, denotes the interval, $kT_I \leq t < (k+1)T_I$, over which the CAF, given by (5), is calculated. To identify the presence of the desired signal this variable is compared to some threshold, $V_T$, which is generally tuned to provide a desired false-alarm probability \[1\] \[21\]. In the presence of interference, as can be seen in Fig. 5, however, it is clear that this simple detection scheme will perform poorly. Nonetheless, the signal is clearly distinguishable and, while it does not represent the global maximum, it does represent a local maximum in the decision space.

Provided $\Delta \tau$ and $\Delta F$ are not too small, relative to $T_I$, the desired signal represents not only a local maximum, but also a sharp peak in the search space. It is useful, therefore, to examine the second partial derivative of $D$ in both the $\hat{\tau}$ and $\hat{f}_D$ dimensions. The signal may then be identified amongst the noise and interference, by choosing a decision variable which represents a large change in curvature of $D$ across the search space. One suitable candidate is the Laplacian of $D$, denoted here by $D^{Lap}$, which, for a discrete function such as the acquisition search space, can be approximated by the four-neighbor Laplacian, given by \[30\]:

![TABLE I](image-url)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>GPS C/A PRN number</td>
<td>1</td>
<td>-</td>
<td>$N_0$</td>
</tr>
<tr>
<td>Noise floor</td>
<td>-204.0</td>
<td>dBW</td>
<td>$F_r$</td>
</tr>
<tr>
<td>Sampling frequency</td>
<td>5.0</td>
<td>MHz (real)</td>
<td>$f_F$</td>
</tr>
<tr>
<td>Intermediate frequency</td>
<td>1.25</td>
<td>MHz</td>
<td>$f_0$</td>
</tr>
<tr>
<td>Coherent integration time</td>
<td>7.0</td>
<td>ms</td>
<td>$T_I$</td>
</tr>
<tr>
<td>Non-coherent combinations</td>
<td>5</td>
<td></td>
<td>$K$</td>
</tr>
<tr>
<td>Doppler bin width</td>
<td>100.0</td>
<td>Hz</td>
<td>$\Delta F$</td>
</tr>
<tr>
<td>Code bin width</td>
<td>0.5</td>
<td>chip</td>
<td>$\Delta \tau$</td>
</tr>
</tbody>
</table>
Note that the factors $\frac{1}{\Delta \tau}$ and $\frac{1}{\Delta f}$ are intentionally omitted from the partial derivatives in an effort to render the metric insensitive to code and Doppler bin width. Fig. 4 depicts the $\mathcal{D}^{Lap}$ for search space depicted in Fig. 3. The peak in this new decision variable, corresponding to the GNSS signal, has two contributing factors: firstly, the curvature in the $\tau$ dimension and the curvature in the $f_D$ dimension, both of which are both proportional to the received GNSS signal power; and, secondly, the peaks in the decision variable which correspond to the interference only, in contrast, are dominated by the curvature in the $f_D$ dimension and are proportional to the received interference power. As can be seen both by examination of Fig. 4 or by evaluation of (9) (see [3]), the curvature of $\mathcal{D}$ in the $\tau$ dimension is quite small, even for high interference powers. As the interference power may be significantly higher than the GNSS signal power, $\mathcal{D}^{Lap}$ may still be dominated by the interfering signal.

These observations suggest another possible decision variable, which considers only the curvature in the $\tau$ dimension. Denoted $\mathcal{D}^\tau$ it is given by:

$$
\mathcal{D}^\tau \left( \hat{\tau}, \hat{f}_D \right) = \left| 2\mathcal{D} \left( \hat{\tau}, \hat{f}_D \right) - \mathcal{D} \left( \hat{\tau} - \Delta \tau, \hat{f}_D \right) + \mathcal{D} \left( \hat{\tau} + \Delta \tau, \hat{f}_D \right) \right|.
$$

(13)

An example of $\mathcal{D}^\tau$ corresponding, once again, to Fig. 3 is presented in Fig. 5. In this case the GNSS signal is clearly identifiable within the search space, confirming that, in the presence of strong interference, the signal is best characterized by a large curvature along the $\tau$ dimension. Indeed, use of this new decision variable has implications for the calculation of a suitable detection threshold and will have a corresponding effect on the achievable detection and false-alarm probabilities. These and other issues will be discussed further in Sections IV and V wherein insights gained here will be employed to develop more robust acquisition decision variables.

Another useful property of the interference corrupted CAF, is that the decision variables are somewhat symmetrical. This is depicted in Fig. 6 wherein the search space has been extended significantly for illustrative purposes. In this case, the interference frequency is 5 kHz below the intermediate frequency and, so, the square magnitude of the value of the interference component, $S_{int}(\hat{\tau}, \hat{f}_D)$, is bilateral-symmetric around the plane $\hat{f}_D = f_x = f_{int} - f_{IF}$ which, in this case is $-5$ kHz. In contrast, neither the GNSS signal nor the additive noise exhibits any such symmetry. If the frequency of the interfering signal is known, therefore, these facts might be exploited in the detection of the GNSS signal. Specifically, the receiver can mitigate the effects of the interfering signal by examining the difference between a given point in the search space and the point given by a projection through the plane of symmetry. This property and the development of a related decision variable will be discussed further in Section IV.

Finally, it is worth commenting here that $S_{int}$ is highly predictable. Given the non-coherent nature of $\mathcal{D}$ and, therefore, any derived decision variables, only the interference frequency and amplitude need to be known to the receiver to estimate the value of $S_{int}$ across the search space. Exploitation of this property for the purposes of

\footnote{For interested readers, this property can be derived by expansion and evaluation of (9), see [3], for details.}
acquisition is discussed further in Section IV-C. As an example, however, Fig. 7 illustrates the search space for a GNSS signal corrupted by a CW interference with a JNR of 10 dB. In this case, the amplitude and frequency of the interference are perfectly known to the receiver and, using (9) and (11), has been removed. Interestingly, the GNSS signal represents the global maximum over the search space. Further details of this procedure will be discussed further in Section IV-C.

IV. CELL-LEVEL DETECTION PERFORMANCE

This section examines the cell-level acquisition performance of the receiver experiencing strong CW interference. The insights gained from Sections I and II are leveraged to develop new decision variables which provide a degree of interference resilience. Environments wherein the receiver has either: no prior knowledge of the interference, knowledge of the interference frequency, or both knowledge of the interference frequency and its amplitude, are considered, respectively, in Sections IV-A, IV-B, and IV-C.

In the absence of interference, the traditional decision variable, \( D \), is compared to a threshold, \( V_T \). The value of \( V_T \) depends, amongst other things, on the desired false-alarm probability, and can be readily calculated given an estimate of the receiver’s noise-floor [1 21 31]. The most practical technique to estimate the noise variance is to correlate the received signal with a local replica of an unused PRN code. This method is reliable as the noise floor is effectively uniform over the entire acquisition search space and does not vary noticeably from one PRN to another. As has been shown in Section III, however, the same is not true in the presence of a CW interference.

Although the relationship between the probability of false-alarm and detection threshold in the presence of interference can be described (see, for example, [3]), the expression is rarely useful for receiver tuning. This reason is that both the interference power and frequency must first be known to the receiver. Therefore in the more likely event that the receiver does not know these interference properties, \( V_T \) cannot be set for the traditional decision variable based on desired probability of false-alarm.

A number of problems become apparent at this point; choosing an appropriate value for \( V_T \) becomes difficult and, moreover, it is likely that the resultant acquisition performance is unsatisfactory. To present the cell-level results in this section, the interference frequency was fixed such that \( f_{\text{int}} - f_{1F} = 125 \text{ Hz} \). Fig. 8 illustrates this problem, wherein an interference power of \(-130 \text{ dBW} \) (JNR of 10 dB) is assumed and the ROC is assessed for the non-quantizing and 1-, 2- and 3-bit quantizing receiver. It is clear that, for any reasonably low \( P_{fa} \) value, regardless of the quantizer configuration, the best attainable \( P_d \) is still effectively unusable. It seems that tuning the traditional acquisition scheme will not suffice. To address this problem, this section presents a selection of novel decision variables, based upon the observations made in Section III.

Similar to traditional schemes, the detection quality of the proposed decision variables is dependent, primarily, upon the \( C/N_0 \), interference power, and quantizer configuration. An increase in \( C/N_0 \) value always improves the acquisition performance to some extent however, in the case where CW interference is present, the acquisition performance is dependent primarily on the interference rather than thermal noise. In the following, to simplify further analysis, the received signal is assumed to have a \( C/N_0 \) of 45 dB-Hz in all of the subsequent results.
A. The Window-Based Acquisition Scheme

When the receiver lacks a priori information regarding the presence or properties of the interfering signals, modifying the traditional search space can prove useful with the modified space, $D^\tau$, showing particular promise. The ROC performance of this decision variable and some of its variations are considered here. It is noted that, although the differencing with adjacent cells provides insensitivity to interference, it also increases the noise component of the decision variable. To alleviate this effect, the $D^\tau$ function can be generalized to consider not only the immediately adjacent cells, but a window surrounding the cell under test. This window-based decision variable can be expressed as:

$$D^{\text{Wind}}(\hat{\tau}, \hat{f}_D) = \left| D\left(\hat{\tau}, \hat{f}_D\right) - \langle D\left(\hat{\tau}, \hat{f}_D\right) \rangle_W \right|$$  \hspace{1cm} (14)

where, the operator $\langle \cdot \rangle_W$ represents the average of the decision variable values in $\tau$ dimension calculated across a window of $W$ cells centered on the cell under test. As can be seen in (14), the average of decision variable values over code delay space is subtracted from the conventional decision variable, $D$, in (11). The second term in (14) can be estimated by considering a small window centered at the cell under test, when this window extends one cell in either direction, it is equivalent to $D^\tau$. The ROC performance of this decision variable is presented in Fig. 9 for both the non-quantizing and the 2-bit quantizing receiver assuming a JNR of 10 dB. The results show that $D^{\text{Wind}}$ can provide a significant improvement over the traditional decision variable, a result that will become even more evident in the system-level analysis in Section V. A dependence on the quantizer resolution is still apparent, in particular for the 1-bit case, and stems from the unavoidable signal-strength losses incurred by jamming effects in the quantizer.

B. The Frequency-Pair Acquisition scheme

If a receiver has some a priori information regarding the prevailing interference frequency, a modified decision variable can be employed. As illustrated in Section III, the interference component of $D$ exhibits bilateral-symmetry around the plane $\hat{f}_D = f_{\text{int}} - f_{IF}$. Thus, a decision variable based on this property can be defined:

$$D^{\text{Pair}}(\hat{\tau}, \hat{f}_D) = \left| D\left(\hat{\tau}, \hat{f}_D\right) - D\left(\hat{\tau}, \hat{f}_{D,\text{Pair}}\right) \right|$$ \hspace{1cm} (15)

$$\hat{f}_{D,\text{Pair}} = 2f_{\text{int}} - 2f_{IF} - \hat{f}_D.$$
The interference component of \( D \), for a given cell under test will be approximately equal to that of the cell centered at the \( \text{pair} \) frequency. Differencing these two cells will leave only the signal, if it is present, the thermal noise and, perhaps, some residual interference power. If the interference frequency is sufficiently well known, this decision variable can significantly improve detection performance, albeit at the cost of increasing both the computational load and decision variable noise by a factor of two. This scheme is insensitive to the true interference amplitude, to the true interference frequency, and to error in the receiver estimate of the interference amplitude. It is, however, sensitive to error in the receiver’s estimate of the interference frequency as it will result in the incorrect pairing of cells.

Fig. 11 demonstrates the cell-level ROC performance of the frequency-pair method for 1-, 2-, 3-bit and non-quantizing receivers for a JNR of 10 dB. As compared to the traditional decision variable, depicted in Fig. 8, this decision variable delivers significant performance improvements. It is evident, however that the performance of this decision variable significantly degrades for the 1-bit receiver. This is due to the introduction of harmonics in the interfering signal by the quantizer, as depicted earlier in Fig. 2. Unlike the fundamental interference frequency, the harmonics are likely to fall outside the Nyquist band and, thus, experience spectral folding and appear elsewhere within the pass-band. Unfortunately, these folded harmonics may no longer exhibit symmetry in the search space. As this symmetry is the very property upon which this decision variable operates, the interference contribution to the search space cannot be readily canceled. An increase from one to two bits of quantizer resolution is sufficient to yield a stark improvement in performance. In fact, interestingly, the 2-, 3-bit receivers appear to slightly outperform the non-quantizing receiver for high \( P_{\text{fa}} \) values.

Fig. 12 illustrates the effect of error in the estimation of the interference frequency on the ROC performance of the scheme based on \( D^\text{PAIR} \) decision variable. A JNR of 10 dB has been assumed and, for the sake of simplicity,
only the 3-bit and non-quantizing receivers have been considered. As may be expected, when the error is small with respect to the coherent integration period the performance degradation is minimal. As can be seen, the ROC performance given a 5 Hz error is minimal. Larger errors, such as 25 or 100 Hz, cause the ROC performance to degrade to a point where the decision variable is unusable. Of course, the tolerable frequency error is inversely proportional to the coherent integration period. It is useful to note here that the sensitivity of the decision variable to frequency error is similar for both the 3-bit and non-quantizing receivers, suggesting it to be relatively independent of the quantizer configuration.

C. Direct Interference Removal

As discussed in Section II-C the interference component of $D$ is highly predictable. Therefore, in the event that the receiver has a priori information about both the interference frequency and its amplitude, its contribution to $D$ can be directly removed. Being a non-coherent metric, $D$ is insensitive to both the phase of the interference and the relative phases of the interference and the GNSS signal. Given knowledge of the quantizer configuration, the AGC gain and the sample rate, the interference contribution, $S_{\text{int}}$, can be evaluated. Alternatively, a synthesized interference can be computed, quantized and correlated with the local replica signal. Subtracting the predicted interference from the traditional decision variable, $D$, produces a new pseudo-interference-free search space, denoted here by $D^{\text{IntR}}$. Fig. 13 shows the ROC performance of a receiver employing this scheme.

Although this procedure does significantly increase detection performance relative to the traditional approach its performance is sensitive to the accuracy of the a priori interference amplitude and frequency estimates. Studies have shown that, via the use of notch filters, FFT or subspace algorithms, these parameters can be estimated to within $\pm 1$ dB and 25 Hz, respectively. As in interference transmitter is, generally, within a small range to the receiver, under even benign dynamics, the amplitude and frequency uncertainties can grow considerably. Illustrated also in Fig. 13 is the ROC performance when the amplitude and frequency estimates this amount (1.0 dB and 10 Hz, respectively). Interestingly, although this sensitivity is low for high $P_{\text{fa}}$ values, but is far more significant for values below $1.0 \times 10^{-2}$, which are of interest for GNSS applications. Moreover, requiring the most a priori information of all decision variables, it still underperforms for the 1-bit case.

Fig. 13. The ROC performance of $D^{\text{IntR}}$ given a JNR of 10 dB for a selection of quantizing and non-quantizing receivers assuming perfect interference power and frequency estimation (solid lines) and for respective power and frequency errors of 1.2 dB and 25 Hz (broken lines).

V. SYSTEM-LEVEL ACQUISITION PERFORMANCE

This section discusses the performance of the traditional and new decision variables presented in Section IV. Although many different search, detection and verification strategies exist, for simplicity, this work considers the parallel search scheme [1] [21][31] employed for the detection of the signal, followed by a binary-integration verification scheme. The initial detection scheme computes the decision variable of choice over the entire code and frequency space and declares the cell corresponding to the largest value as likely containing the GNSS signal. The presence, or otherwise, of the signal is subsequently verified by comparing the value of the decision variable calculated based on the cell coordinates, to a threshold, over one or more successive periods. Here, the performance of this system-level scheme is evaluated for each of the four decision variables discussed in Section IV namely the traditional, window-based, pair-based and interference-removing decision variables.

Monte-Carlo simulation analysis has been used to evaluate the acquisition performance for each of the four decision variables for each of the non-quantizing, 1-, 2- and 3-bit quantizing receivers. Both the interference frequency and signal Doppler were randomly assigned in the range $f_{1F} \pm 5$ kHz and a range of interference power levels in the range $-150$ to $-90$ dBW. For each of these scenarios, the acquisition performance was evaluated by considering a selection of received signal configurations, corresponding Doppler values in the range $\pm 5.0$ kHz and
all code phase values. Each simulation configuration was then repeated \(10^5\) times. The results of this simulation campaign are presented in Fig. 14 in terms of detection probability versus interference power.

These results illustrate a stark performance improvement for some of the modified decision variables, relative to the traditional approach, however, it appears that some are more suited to certain receiver configurations than others. In particular, some decision variables appear to be particularly sensitive to quantization effects.

Rather unsurprisingly, it is clear that the scheme which utilizes the most a priori information provides the best performance. Examining Fig. 14 (a), it can be seen that the interference removing scheme can sustain CW interference with and interference power of the order of \(-100\) dBW without significantly degrading the detection performance. The effects of quantization however, are significant, resulting in a reduction in the performance at a significantly lower power. For example, the use of a 3-bit quantizer reduces the tolerable interference power by 15 dB, and each of the 1- and 2-bit quantizers incur a further reduction of approximately 5 dB, as evidenced in Figs. 14 (b), (c) and (d).

While relaxing the requirements for a priori information, the pair-based acquisition scheme can provide quite impressive performance in the presence of interference. In the case of the non-quantizing receiver, it provides almost as good detection performance as the interference removing scheme. Unfortunately, it suffers significantly from the effects of quantization, as can be seen by the dramatic drop in tolerable interference power from Fig. 14 (a) to (b), of more than 30 dB. This trend continues with reducing quantizer resolution, resulting in a detection performance for the 1-bit receiver that is significantly worse than the traditional decision variable.

Considering the most likely receiver operating scenario, where it has no a priori information whatsoever, the window-based detection scheme appears to provide quite significant resilience to interference. Although in the non-quantizing receiver scenario, it fails to match the performance of the pair-based and interference-removing schemes, for all quantizing receiver cases, it compares well. In comparison to the traditional acquisition scheme, it provides from anywhere from 15 to 20 dB improvement in the tolerable JNR and, even for the 1-bit quantizing receiver, can provide a probability of detection in excess of 0.9 in the presence of a CW interference signal with a power of approximately \(-130\) dBW. Moreover, in the case that the receiver performs FFT-based correlation, this performance enhancement is delivered at no extra correlation cost to the receiver and without the need for any a priori information.

Subsequent to the detection of a candidate cell, the presence of the signal must be verified by comparing the value of the decision variable to a threshold. In the absence of interference this is a relatively simple task: the receiver generally maintains an accurate estimate of the thermal noise contribution to the IF signal and can assign an appropriate threshold value given the inverse cumulative density function of an appropriate chi-square distribution. In the presence of a strong interference signal, however, this task is complicated. The appropriate threshold for a traditional decision variable depends not only on the thermal noise floor, but also on the interference frequency and power. For the decision variables presented here, however, the case is much simplified, as depicted in Fig. 15. The traditional decision variable exhibits a broad, non-zero mean distribution, both in the presence and absence of the GNSS signal. In contrast, the new decision variables exhibit a narrow, approximately zero mean distribution in the absence of the signal and a broad non-zero mean distribution in its presence.

The fact that the distributions of the three new decision variables are approximately zero mean makes them particularly suited to use in a binary-integration scheme. Any choice of threshold greater than zero will ensure a false-alarm probability of 0.5 or less, regardless of the noise or interference powers. Thus, by choosing coincidence parameters appropriately, a lower bound can be imposed on the false alarm rate. As a simple example, choosing \(M = N = 10\) will ensure a bound of \(P_{fa} < 1.0 \times 10^{-3}\) for any \(V_T \geq 0\). Moreover, being computed as the difference of two non-central chi-square distributions, they can be reasonably well approximated by a Laplace, or double-exponential distribution [35].
Assuming, therefore, that the false alarm probably is bounded to a reasonable value, then even a coarse estimate of the variance of the decision variables under $H_0$ can suffice to select a threshold. If this variance of the decision variables is given by $\sigma^2$, then a suitable threshold can be found via:

$$V_T = -\frac{\sigma}{\sqrt{2}} \log(2P_{fa})$$

(16)

where $P_{fa}$ is the target false-alarm rate. It is worth stressing, of course, that the value of $V_T$ is less critical for these new decision variables, than for the traditional one as, provided $V_T > 0$, then $P_{fa}$ is bounded by the choice of coincidence parameters. The relative performance of the various decision variables as used in an M-of-N verification strategy is presented in Figure 16.

The verification strategy was configured with $M = 3$, $N = 4$, assuming integration parameters as per Table I, and a threshold calculated via (16) using $P_{fa} = 0.001$, and an estimate of $\sigma$ computed from the preceding detection search space. As can be seen, the approximation of Laplace distribution offers reasonable detection performance. Evident also is the significant improvement that can be made over the traditional decision variables, where the interference-removing and pair-based schemes can accommodate 4 dB higher interference power levels, and the window-based scheme tolerates 8 dB higher interference power levels.

VI. CONCLUSIONS

The performance of the acquisition process for a typical consumer GNSS receiver in the presence of continuous wave interference is examined in this paper. The limitations of the traditional acquisition processor is shown and three new alternative approaches are examined.

Through an analysis of the effects of interference on the observed acquisition search space novel decision variables, which provide robustness against this
interference, are developed, considering both the operation of a traditional-receiver, and one which may have knowledge of the interference characteristics. The resultant acquisition performance enhancement is examined at the cell-level, in a parallel detection scheme and coincidence-based verification scheme, wherein an approximate threshold selection criteria is presented. Results show that these novel decision variables can provide a significant improvement in system-level detection performance. In particular, the window-based scheme emerges as a particularly effective, robust and efficient method and can provide acceptable detection performance in the presence of higher interference power levels than that can be tolerated by traditional acquisition schemes.

REFERENCES


Mohammad Abdizadeh received his M.Sc. degree in Microelectronics Engineering in 2008 from Sharif University of Technology, Iran and, his Ph.D. in Geomatics Engineering in 2013 from the Department of Geomatics Engineering, University of Calgary, Canada. His current research interests includes signal processing, adaptive filtering and, GNSS signal acquisition and tracking.

James T. Curran was born in Cork, Ireland in 1984. He received a B.E. in Electrical & Electronic Engineering in 2006 and a Ph.D. in Telecommunications in 2010, from the Department of Electrical and Electronic Engineering, University College Cork, Ireland. He worked as a senior research engineer with the PLAN group in the University of Calgary from 2011 to 2013 and is currently a Grantholder at the European Commission, Joint Research Center in Ispra, Italy. His main research interests are signal processing, information theory, cryptography and software defined radio for GNSS applications.

Gérard Lachapelle (M’81) holds a Canada Research Chair in wireless location in the Department of Geomatics Engineering, University of Calgary, Calgary, AB, Canada. He has been a professor at the University of Calgary, Calgary, AB, Canada, since 1988, and is now member of the PLAN Group. He has been involved in a multitude of Global Navigation Satellite Systems (GNSS) R&D projects since 1980, ranging from RTK positioning to indoor location and GNSS signal processing enhancements. Professor Lachapelle has received numerous awards for his accomplishments.