Combined Spatial-Polarization Correlation Function for Indoor Multipath Environments

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Abstract—The combined spatial-polarization correlation function for indoor multipath environments is discussed. An analytical model based on an isotropic receiving antenna and a sphere of scatterers is developed for quantifying signal decorrelation as a function of antennas’ spatial and orientation separation. A closed form expression is deduced for the combined spatial-polarization correlation function which is useful for quantifying the potential diversity gain in antenna diversity systems. Finally, an extensive set of measurements were carried out at 1947.5 MHz to verify the theoretical results.

I. INTRODUCTION

Spatial and polarization diversity techniques have proven effective in mitigating multipath fading and have been in use in wireless communications for several years. Spatial diversity systems are established based on the fact that in a multipath fading environment, received signals on the diversity branches decorrelate spatially where the rate of decorrelation increases monotonically with enlarging the multipath angular spread [1]. Polarization diversity based on antennas with different polarization characteristics is another antenna diversity technique and is known to be nearly as effective as spatial diversity [2]-[4]. In a polarization diversity system, correlation between signals of two similar co-located antennas decreases by increasing the angular separation of antennas’ polarization vectors such that antennas with orthogonal polarizations are considered as uncorrelated diversity branches[2],[3].

In order to quantify the achievable diversity gain in an antenna diversity system, correlation between diversity branches is required. A theoretical model for characterizing signal decorrelation versus antennas’ spatial separation was first discussed by Clarke [5]. Clark’s model, which is based on a dense ring of scatterers in the azimuth with the receiving antenna near the center of the ring, results in a simple closed form expression for the correlation function which is in the form of a Bessel function. A modification to Clark’s model is discussed by [6] based on a continuous sphere of scatterers and is more suitable for indoor multipath environments since indoors is more similar to an enclosed cavity rather than an azimuthal ring of scatterers.

A theoretical model for characterizing correlation between diversity branches in a polarization diversity system at base station was first established by Kozono et. al. [7] which assumes a narrow multipath beam arriving from the azimuth. Vaughn [8] further extends Kozono’s model to account for the rotation of two antennas with a fixed angular separation around their phase centre. As stated earlier, both Kozono and Vaughan models assume a very narrow multipath beam and therefore mostly apply to base-station diversity systems. Correlation between orthogonal components of the electromagnetic field at the mobile is also discussed by [9] which is based on a uniform but arbitrarily wide angular spread. More recently, Brown [10] established a model to account for an arbitrary distribution of multipath power in azimuth and elevation at the mobile. Nevertheless, Brown’s model is not in closed form and involves multiple-fold integrations. In addition, Brown assumes two orthogonal polarizations (x and y) in the Cartesian coordinates while with a spherical distribution of scatterers (typical of an indoor multipath scenario) three polarizations need to be taken into account. Another model based on three field polarizations is considered by [11] which is based on a ring of scatterers and therefore does not apply to indoor environments.

With the increasing interest in utilizing joint spatial and polarization diversity at the mobile (e.g. [12]), quantifying the combined spatial-polarization diversity at the mobile is necessary. Although generalized formulation for evaluating antenna correlation is given [13], there are few analytical models that result in closed form expressions and to the extent of author’s knowledge are discussed either for spatial diversity or polarization diversity systems except for [14] that proposes an empirical equation based on a set of measurements in a reverberation chamber.

This paper develops an analytical model based on a sphere of scatterers and derives a simple closed form expression for the combined spatial-polarization correlation function based on an isotropic antenna.

II. THEORETICAL ANALYSIS

A uniform sphere of scatterers is assumed. Although, other distributions of power in azimuth and elevation can be considered, e.g. [10] & [11], sphere of scatterers is a mathematically simple model that results in a closed form expression for the correlation function and is recognized to
V. Dehghanian, J. Nielsen, and G. Lachapelle characterize multipath in indoor environments at the mobile [6], [12]. Following Collin and Zucker, [15], the open-circuit voltage, \( V \), induced at an antenna output can be found as

\[
V_i = \left[ \mathbf{E}_i(\Omega) \cdot \mathbf{E}_i(\Omega) \right] d\Omega
\]

(1)

where, \( \Omega \) is the solid angle \((\theta, \phi)\), \( \mathbf{E}_i(\Omega) d\Omega \) is the incident electric field from sources arriving from a solid angle \( d\Omega \) at the receiving antenna, \( \mathbf{E}_i(\Omega) = \mathbf{E}_i(\theta, \phi) + \mathbf{E}_i' \mathbf{\phi} \) is the vector effective length of antenna and is a quantity related to the antenna’s far-zone radiated electric field [16].

Source polarizations \( E_{\alpha_i}, \; E_{\phi_i} \) are assumed to be identical, uncorrelated and each spatially white. Also all of the sources are assumed to be uniformly distributed on a spherical surface in the far-field of the receiving antenna (sphere of scatterers model). Consequently, it can be shown that \( V_i \) is a zero-mean complex Normal random variable, \( \text{CN}(0, \sigma^2) \), where \( \sigma^2 \) is the variance and is related to the received signal power at the mobile antenna [17].

The goal is to evaluate the correlation \( \rho(p, \Psi) \) between the open circuit voltages of two identical antennas I and II with spatial-polarization separation of \( (p, \Psi) \) where \( p = |\mathbf{P}_1 - \mathbf{P}_II| \), \( \mathbf{P}_I, \mathbf{P}_II \) are the position vectors of the antennas, and \( \Psi \) is the angular separation between antennas’ polarization vectors (see Fig. 1). The mutual coupling between the antennas is assumed to be negligible. Also, it is assumed that the antennas have perfect linear polarizations with a zero cross polarization discrimination (XPD). Note that in reality the XPD is not zero however, moderately low XPD can be obtained in practice.

The effect of non-zero XPD is the increase in signal correlation between polarization diversity branches and a detailed discussion can be found in [11].

Due to the spherical symmetry of the sources, and without any loss of generality we assume that both antennas are located on the x-z plane of the Cartesian coordinate system. Consequently,

\[
\rho(p, \Psi) = \frac{\langle V_{II}^* V_{II} \rangle}{\sqrt{\langle V_{II}^* V_{II} \rangle \langle V_{II}^* V_{II} \rangle}}
\]

(2)

where \( \langle \ldots \rangle \) is an expectation operation that can be replaced by a time average based on the approximation of signal Ergodicity which results from the approximate stationarity of the environment during the course of measurements.

Consequently and following [8]

\[
\langle V_{II}^* V_{II} \rangle = \int \mathbf{E}_I(\Omega) \cdot \mathbf{E}_I(\Omega) d\Omega \int \mathbf{E}_{II}(\Omega) \cdot \mathbf{E}_{II}(\Omega) d\Omega
\]

(3)

Taking the expectation inside the integrals and considering that \( \mathbf{E}_\alpha \) is spatially white i.e.

\[
\langle E_{\alpha_i}(\Omega) E_{\alpha_j}(\Omega) \rangle = \langle E_{\alpha_i}(\Omega) E_{\alpha_j}(\Omega) \rangle = c \delta(\Omega - \Omega)
\]

one obtains

\[
\langle V_{I}^* V_{II} \rangle = c \int \left[ \mathbf{E}_{II}^* E_{II}^* + \mathbf{E}_{II}^* E_{II}^* \right] \delta(\Omega - \Omega) d\Omega d\Omega
\]

(4)

where \( c \) is a constant related to the amplitude of the far-zone electric fields and will be normalized throughout the rest of the paper.

Now, consider a linearly polarized antenna with isotropic pattern and a vector effective length of

\[
\mathbf{E}_I = \hat{\theta} e^{j \lambda p}
\]

(5)

where \( k = -k (\hat{x} \sin \theta \cos \phi + \hat{y} \sin \theta \sin \phi + \hat{z} \cos \phi) \) is the vector wavenumber with \( k = 2 \pi / \lambda \) and \( \lambda \) the carrier wavelength.

Consider antenna-II with the position vector, \( \mathbf{P}_II \), and the angular separation of \( \Psi \) with respect to antenna-I as shown in Fig. 1. Consequently the vector effective length of antenna-II can be shown to be

\[
\mathbf{E}_{II} = \left( \hat{\theta} \cos \psi + \hat{\phi} F(\theta, \phi) \right) e^{j \lambda p}
\]

(6)

where \( F(\theta, \phi) \) is nuisance function and will be eliminated throughout the process. By replacing (5) and (6) in (2) and by taking (4) into account one obtains

\[
\rho(p, \Psi) = \frac{\cos \Psi}{4 \pi} \int_0^{2 \pi} \int_0^\pi e^{-ikp \sin \theta \pm \Psi} \sin \theta \, d\theta \, d\phi
\]

(7)

where \( \Delta \Phi = \mathbf{P}_I - \mathbf{P}_II \).

Due to the spherical symmetry of the scatterers the result cannot be a function of \( \theta_p, \phi_p \). Therefore, we let \( \theta_p = 0 \) and consequently,

\[
\rho(p, \Psi) = \sin(kp) \cos \Psi.
\]

(8)

Equation (8) provides a simple closed form expression for the spatial-polarization correlation function of a linearly polarized and isotropic antenna. Equation (8) can be used to characterize the diversity gain of a combined spatial and polarization diversity system in an indoor multipath environment. Also, as can be seen from (8), the combined spatial-polarization correlation function is a separable function of space, \( p \), and orientation, \( \Psi \), variables.

### III. EXPERIMENTAL RESULTS

Pilot CDMA signal transmitted at 1947.5 MHz was captured by a half-wavelength dipole in an indoor non-line-of-sight environment. The antenna was connected to an RF front-
end receiver for down-conversion and sampling at 10 MHz. A non-metallic rotary arm ran by a stepper motor was mounted on a precise linear motion table moving with a constant linear speed of \( v = 5 \) mm/s. The rotary arm was set to rotate the dipole antenna with a constant angular speed of \( \omega = \pi/2 \) rad/s. The experimental setup and the measurement equipment are shown in Fig. 2. The magnitude of correlation coefficient of output signal of the rotating antenna was then calculated and plotted in Fig. 3. Note that both spatial and angular separations are functions of time such that \( \Delta(t) = \omega t \) (rad) and \( \rho(t) = vt \) (m) which \( \omega \) and \( v \) were given earlier. As can be seen, the experimental results are in good agreement with the theoretical predictions of (8).

Note that the measured decorrelation based on the antenna rotation is not as high as expected which is due to the fact that the XPD of the dipole antenna with physical dimensions is not zero. In addition, scattering from the nearby objects is another factor that affects the measured correlation.

### IV. CONCLUSIONS

An analytical model based on a sphere of scatterers was developed for characterizing the combined spatial-polarization correlation function of a linearly polarized antenna in an indoor environment. An isotropic antenna was considered for the analysis which resulted in a simple closed form expression for the combined spatial-polarization correlation function. It is evident from the analytical results that the correlation function is separable as a function of space and polarization separation. The analytical results were validated through an extensive set of measurements and a good agreement between the measurements and the theoretical predictions was observed.

### REFERENCES


