MERGER OF POLARIZATION AND SPATIAL DIVERSITY BY MOVING A PAIR OF ORTHOGONALLY POLARIZED DIPOLES

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ABSTRACT

Size and cost considerations of handheld terminals limit the use of antenna array for combating multipath fading in these applications. Recently, a synthetic array based on moving a single antenna was proposed for handheld terminals. This technique however fails to provide enough diversity gain when the channel is spatially correlated. A combination technique based on merging polarization and spatial diversity and motions of a crossed dipole is proposed here. The diversity performance of the crossed dipole synthetic array (XSA) is then compared to that of a single dipole synthetic array. A closed form expression is also derived for calculating the cumulative distribution function of summation of correlated central chi-squared random variables with four degrees of freedom which arises in the XSA performance analysis. Finally, the proposed technique is put into test through an extensive set of measurements.

Index Terms— Diversity, polarization, array, square-law combining

1. INTRODUCTION

Diversity combining is an effective technique for mitigating signal fading, which is a typical of most indoors and terrestrial-outdoors wireless channels. Diversity techniques are based on combining uncorrelated or partially correlated signals in order to decrease the probability of occurrence of deep fades in a multipath fading environment. Most of the work on diversity combining is based on independent diversity branches and the results are well documented in classic works [1][2].

Antenna diversity is a powerful means for obtaining uncorrelated and/or partially correlated diversity branches through employing multiple antennas with different radiation characteristics, different orientations, and different positions [3]. Antenna arrays with a sensor separation of ten or more wavelengths are in wide spread use at base stations [4][5]. Even though the required element spacing at base stations is large due to the small incoming-wave angle spread, it can be shown that in severe multipath environments with multipath angle spread of 2π, such as indoor channels, the required antenna separation can be as small as half a wavelength [4][5][6][7]. Even with this small element spacing, the size of a multi-element antenna array is not compatible with the small size of a handheld terminal and therefore spatial diversity through an array of spatially separated antennas is not practical for the handheld terminal applications. Recently, [8] has proposed a novel technique to achieve spatial diversity in the handheld terminals based on motions of a single antenna in order to form a spatially distributed synthetic array. Although the concept of synthetic array for signal parameter estimation has been around for several years [9], its application for combating multipath fading was first introduced by [8]. As stated earlier, the technique proposed by [8] is based on the motion of a single antenna in a severe multipath environment which facilitates collecting uncorrelated spatial samples every half a wavelength. These uncorrelated samples can be later combined for achieving diversity gain. Nevertheless, for scenarios with smaller (<2π) multipath angle spreads, a pedestrian user requires to move longer distances (due to the longer channel coherence length) in order to obtain the minimum required number of uncorrelated samples. To overcome this problem, this paper proposes a combination technique based on merging polarization and spatial diversities in a synthetic array antenna as shown in Figure 1.

Polarization diversity is another antenna diversity technique and is known to be nearly as effective as spatial diversity [3][5][8][10]. Polarization diversity based on two orthogonally polarized antennas does not require any spatial separation between antenna elements, which makes it more interesting for handheld terminal applications.

This paper evaluates the achievable detection enhancement arising from adding a cross-polarized antenna to the synthetic array. Section 2 defines the system model and assumptions. Section 3 discusses the realizable detection enhancement through comparing the performance of the proposed XSA with that of a single antenna synthetic array (SSA) in a generalized correlated Rayleigh fading. Section 4 discusses the measurement results. Conclusions are given in Section 5.

2. SYSTEM MODEL AND ASSUMPTIONS

Multipath fading at the output of a standard dipole in severe and isotropic multipath environments such as indoors...
can be modeled according to the “Sphere of Scatterers Model” [6][11][12]. According to this model, the magnitude of the complex correlation coefficient, \( |\rho| \), between complex baseband voltages at the output of two co-polarized isotropic antennas with spatial separation of ‘d’ follows from [11][12] as

\[
|\rho| = \left| \frac{\sin(2\pi d / \lambda)}{2\pi d / \lambda} \right| \quad (1)
\]

where \( \lambda \) is the carrier wavelength. Therefore, by moving an antenna along an ‘M/2’ long smooth trajectory in a multipath fading environment, one can collect up to \( M \) uncorrelated signal samples [8]. Let us assume that these \( M \) signal samples resemble \( M \) diversity branches experiencing flat Rayleigh fading\(^1\). The entire duration of signal sampling or the signal snapshot is denoted by \( T \). The signal snapshot period is assumed to be divided into \( M \) equal sub-intervals, each \( \Delta T \) seconds long, i.e. \( T = M \Delta T \). The communication channel is assumed to be stationary during each sub-interval. Based on the Rayleigh fading assumption, multipath rays impinging on the antenna during the \( i \)-th sampling sub-interval \( (i-1)\Delta T \leq t < i\Delta T \) induce a baseband voltage at the antenna output that is denoted by

\[
r_i = s_i + w_i \quad \text{for} \quad i = 1, \ldots, M \quad (2)
\]

where \( \mathbf{S} = \begin{bmatrix} \mathbf{S}^{(p)} & \mathbf{S}^{(o)} \end{bmatrix} = \begin{bmatrix} s_i^{(p)} & \ldots & s_M^{(p)} & s_i^{(o)} & \ldots & s_M^{(o)} \end{bmatrix} \) are jointly Gaussian complex random variables (RVs) distributed according to \( \mathbf{S} \sim CN(\mathbf{0}, \mathbf{C}_s) \) with \( CN(\mu, \mathbf{C}_s) \) denoting a complex Gaussian distribution with \( \mu = E(\mathbf{S}) \), \( C_s = E[(\mathbf{S} - \mu)(\mathbf{S} - \mu)^\dagger] \) and ‘E’ representing the expected value operation. Also, ‘\( \dagger \)’ denotes a Hermitian, and ‘\( \dagger \)’ denotes a matrix transpose operation. In addition, the superscripts ‘(p)’ and ‘(o)’ stand for the principal and the orthogonal polarizations, respectively. Furthermore, it is assumed that the baseband output signals of the orthogonally polarized antennas are uncorrelated and identically distributed. In other words

\[
s_i^{(p,o)} \sim CN\left(0, \sigma_i^2 / M \right) \quad (3)
\]

and

\[
\left\{s_i^{(p)}, s_i^{(o)}\right\} = 0 \quad (4)
\]

where \( \sigma_i^2 / M \) is the signal variance after \( \Delta T \) seconds of coherent integration. Furthermore, \( \mathbf{W} = \begin{bmatrix} w_i^{(p)}, \ldots, w_M^{(p)}, w_i^{(o)}, \ldots, w_M^{(o)} \end{bmatrix} \) represents the independent Gaussian noise distributed according to \( CN(\mathbf{0}, \mathbf{C}_w) \) where \( \mathbf{C}_w = \sigma_w^2 \mathbf{I}_{2M} \) represents a noise covariance matrix and \( \mathbf{I}_{2M} \) denotes a 2M by 2M Identity matrix. Noise is assumed to be solely intrinsic and independent of the signal. Note that the initial steps of spread spectrum modulation and demodulation are omitted for the simplicity of expression. Consequently, the baseband complex signal \( s_i \) at the output of a linearly-polarized antenna at the \( i \)-th spatial point is

\[
s_i = m_i \left( c \cdot e^{j\psi_i} \right) \quad \text{for} \quad i = 1, \ldots, M \quad (5)
\]

where \( j = \sqrt{-1} \), \( \psi_i \) is the signal phase, \( c \) is the signal envelope after local mean removal (the fast fading part of the signal), and \( m_i \) represents the local-envelope-mean (the slow-fading part of the signal) which is assumed to be constant \( m_i = cte. \) \( i = 1, \ldots, M \) over the course of measurements. In general, \( m_i \neq m_k \) for \( i \neq k \) and varies slowly due to path loss and shadowing [14]. Nonetheless, the received signal must be normalized to its envelope local-mean in order to filter-out the slow-fading component of the signal [10] [14] [15]. This normalization is essential since it maps the received signal samples into samples of an Ergodic process and consequently validates the use of time-average instead of expected value operator.

### 3. DETECTION PERFORMANCE ANALYSIS

In spread spectrum communication systems, signal acquisition is usually accomplished through a multi-hypothesis search over the unknown parameters such as frequency offset, code offset, etc [16], [17]. In general, the acquisition algorithm is designed based on the target values of probabilities of detection \( (P_D) \) and false alarm \( (P_F) \) [8]. Consequently, the problem is modeled as one of choosing between \( H_0 \), the noise-only hypothesis, and \( H_1 \), the signal present hypothesis where

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1. There is some loss in generality by ignoring the resolvable multipath and flat fading. Indeed, the frequency selective fading results in alternate source of diversity which can be considered in processing gain evaluations.
\[ r_i = w_i, \quad : H_0 \]
\[ r_i = s_i + w_i, \quad : H_1 \]

Accordingly, the measured \( P_F \) and \( P_D \) quantify the performance of the adopted diversity combiner (ex. SLC, selection combiner (SC), Estimator Correlator (EC), etc.) through

\[ P_F (\gamma) = 1 - \mathcal{F}(\gamma) \bigg|_{H_0} \]
\[ P_D (\gamma) = 1 - \mathcal{F}(\gamma) \bigg|_{H_1} \]

where \( \mathcal{F} \) is the cumulative distribution function (CDF) of the combiner output under \( H_0 \) or \( H_1 \) and \( \gamma \) is the threshold [18].

Any target detection performance requirements, \( \{P_{F0}, P_{D0}\} \), maps into a specific required average SNR denoted by \( \eta = \sigma_s^2 / \sigma_n^2 \). Consequently, the corresponding values of average SNR determined for the XSA \( (\eta_x) \) and for the SSA \( (\eta_s) \) will be compared in order to provide a measure of performance enhancement. A quantitative metric can be given by

\[
G_{[P_{F0}, P_{D0}]} = 10 \log_\frac{\eta_x}{\eta_s} \text{ (dB)}.
\]

### 3.1. Choice of Combiner

It is well known that the Estimator Correlator (EC) is the optimum detector in the Neyman Pearson (NP) sense if the signal and noise statistics are zero-mean jointly Gaussian [18]. According to the EC formulation,

\[
z_{EC} = \mathbf{R}^H \mathbf{C}_s (\mathbf{C}_s + \sigma_n^2 \mathbf{I}_M)^{-1} \mathbf{R}, \quad \text{where} \quad \mathbf{R} = \mathbf{S} + \mathbf{W}
\]

and \( \mathbf{R} = [\mathbf{R}^{(p)}, \mathbf{R}^{(o)}] = [r^{(p)}_1, ..., r^{(p)}_M, r^{(o)}_1, ..., r^{(o)}_M] \). As shown in the appendix I, the formulation can be further simplified by noting \( \langle s^{(p)} s^{(o)} \rangle = 0 \) as

\[
z_{EC} = \left[ \mathbf{R}^{(p)} \right]^H \mathbf{C}_s^{(p)} \left( \mathbf{C}_s^{(p)} + \sigma_n^2 \mathbf{I}_M \right)^{-1} \left[ \mathbf{R}^{(p)} \right] + \left[ \mathbf{R}^{(o)} \right]^H \mathbf{C}_s^{(o)} \left( \mathbf{C}_s^{(o)} + \sigma_n^2 \mathbf{I}_M \right)^{-1} \left[ \mathbf{R}^{(o)} \right]
\]

where \( \mathbf{C}_s^{(p)} \), \( \mathbf{C}_s^{(o)} \) denote signal covariance matrices corresponding to \( \mathbf{S}^{(p)} \) and \( \mathbf{S}^{(o)} \), respectively. As can be seen from Equation (9), EC works independently on orthogonal polarizations.

While non-optimum but popular combiners such as the square-law combiner (SLC) provide comparable performance with acceptable simplicity, EC’s practicality in signal detection is rather limited due to its integrated complexities such as signal covariance matrix estimation. It was shown in [8] that only a marginal improvement in signal detection is realized through employing EC as opposed to SLC for small and moderate values of branch correlation. The EC expression in (9) reduces to the SLC formulation when the branch correlation is ignored. In other words

\[
z_{SLC} = \mathbf{R}^H \mathbf{R} = \sum_{i=1}^M \left| r^{(p)}_i \right|^2 + \mu \sum_{i=1}^M \left| r^{(o)}_i \right|^2
\]

where \( \mu = 0 \) corresponds to SSA and \( \mu = 1 \) to XSA scenarios.

#### 3.1.1 SLC Performance in Correlated Rayleigh Fading Channel

Most of the work on SLC is based on independent diversity branches and the results are well documented in the literature [1][19]. Nevertheless, in many real scenarios the fading statistics between diversity branches are correlated requiring analysis of SLC performance with correlated diversity branches. The SLC output statistics for \( \mu = 1 \) (XSA) can be shown to be

\[
\mathcal{F}_{SLC} (\gamma) = \int \int \sum_{i=1}^M \frac{1}{1 - j\lambda \omega} \exp \left(-j\lambda \omega \right) d\omega
\]

where \( \lambda = \lambda^{(o)} = \lambda^{(o)} \) are the eigenvalues of the covariance matrices, \( \mathbf{R}^{(p)} \) and \( \mathbf{R}^{(o)} \) \( (\lambda^{(p)} = \lambda^{(o)} \) arises from the identical distribution of the signal in either polarizations). Note that \( \mathbf{C}_s = \mathbf{C}_s + \mathbf{C}_w \) under \( H_1 \) and \( \mathbf{C}_s = \mathbf{C}_s \) under \( H_0 \). As shown in Appendix II, if the nonzero Eigenvalues of \( \lambda \) are distinct, a simple closed form expression can be derived for the CDF of SLC output, using the partial fraction expansion as

\[
\mathcal{F}_{SLC} (\gamma) = 1 - \sum_{i=1}^M \left( A_i (1 + \frac{\gamma}{\lambda_i}) + B_i \exp \left(-\frac{\gamma}{\lambda_i} \right) \right)
\]

where

\[
A_i = \prod_{k=1, k \neq i}^M \left( \lambda_i / \lambda_k - 1 \right)
\]

and

\[
B_i = A_i \sum_{m=1}^M \lambda_i \lambda_i - \lambda_m
\]

On the other hand, the SLC output statistics for \( \mu = 0 \) (SSA) can be shown to be

\[
\mathcal{F}_{SLC} (\gamma) = \int \int \sum_{i=1}^M \frac{1}{1 - j\lambda \omega} \exp \left(-j\lambda \omega \right) d\omega
\]

"IEEE Canadian Conference on Electrical and Computer Engineering, CCECE 2010, May 3-5, Calgary, AB, Canada"
where $\lambda_i$ are the Eigenvalues of $\mathbf{C}_R$. Similar to the XSA scenario, if the nonzero Eigenvalues $\lambda_i$ are distinct, a simple closed form expression can be derived for the CDF of the SLC output as \[ \mathcal{F}_{\text{SLC}}(\gamma) = 1 - \sum_{i=1}^{M} T_i \exp\left(-\frac{\gamma}{\lambda_i}\right) \] (16)

where
\[ T_i = \prod_{k=1}^{M} \frac{1}{1 - \lambda_i / \lambda_k}. \] (17)

3.1.2. SLC Performance in Independent Rayleigh Fading Channel

As stated earlier, when the diversity branches are uncorrelated, SLC makes the optimum combiner. Observing the statistics of SLC with uncorrelated branch variables provide a better insight to the synthetic array problem due to the simplicity of mathematical expressions. Specifically, the advantage of the XSA model over the SSA model can be simply verified by noting the achievable extra degrees of freedom at the SLC output which will be discussed shortly.

Through moving a single linearly-polarized antenna along a smooth $\lambda/2$-long trajectory during one signal snapshot, one set of $M$ spatially uncorrelated samples can be collected. These samples can be combined using an SLC combiner, whose output statistics is
\[ z_{\text{SLC}} = \chi^2_{2M} \left( 0, \frac{\sigma_s^2 + M \sigma_n^2}{2M} \right) \] (18)

where $\chi^2_N(s^2, \sigma^2)$ denotes a chi-squared distribution with $N$ degrees of freedom ($N$ DOF) with non-centrality parameter $s^2$ and the common variance of the corresponding Gaussian components $\sigma^2$. On the other hand, translating a doublet of orthogonally polarized antennas over the same trajectory provides two orthogonal sets of $M$ spatially uncorrelated samples. The SLC output statistics for this setup is
\[ z_{\text{SLC}} = \chi^2_{4M} \left( 0, \frac{\sigma_s^2 + M \sigma_n^2}{2M} \right). \] (19)

The extra DOF realized for the XSA setup is a simple evidence of performance improvement. This is demonstrated though measured and simulated graphs in the next section.

4. MEASUREMENTS AND ANALYSIS

Two dipole antennas were placed orthogonally to each other to form an array of two crossed dipoles. This doublet was translated along a linear trajectory with a constant speed of 0.5 m/s in an indoor multipath environment. The CDMA signal transmitted from a nearby base-station was then captured by the antennas and was fed into the front-end receiver for down conversion and sampling at 10 MHz. The experimental setup is shown in Figure 2. Figure 3 shows the measured magnitude of complex correlation coefficient for each antenna, which can be seen to be in a good agreement with the theoretical predictions of Equation (1). Furthermore, the magnitude of the complex correlation coefficient between the orthogonal dipole signals was measured as $|\rho| = 0.09$, which is in a good agreement with the assumption made in (4).

The measured ROC curves of Figure 4 provide a visual measure of performance improvement realized through employing an XSA versus an SSA in a spatially correlated channel. In addition, Figure 5 demonstrates the measured and theoretical processing gains (Eq. 8) as a function of $M$ with a sample separation of $d = \lambda/3$. As can be seen from this figure, when the number of spatial samples is small, the XSA configuration considerably outperforms the SSA configuration. Note that the realizable net processing gain (G) decreases as $M$ increases as shown in Figure 5 and Figure 6, which is due to insignificance of extra degrees of freedom for large $M$. 
form expression for the output CDF of summation of correlated central chi-squared RVs with 4 DOF arising in XSA performance analysis was derived based on the partial fraction expansion.

**APPENDIX I**

The EC formulation follows from [18]

$$z_{EC} = R^H C_s (C_s + \sigma_0^2 I_{2M})^{-1} R.$$  

(20)

Following the assumption in Eq. (4), the signal covariance matrix $C_s$ reduces to

$$C_s = \begin{bmatrix} C_s^{(p)} & 0 \\ 0 & C_s^{(o)} \end{bmatrix}.$$  

(21)

Consequently

$$(C_s + C_o)^{-1} = \begin{bmatrix} C_s^{(p)} & 0 \\ 0 & C_s^{(o)} \end{bmatrix} + \sigma_0^2 I_{2M}^{-1} = \begin{bmatrix} (C_s^{(p)} + \sigma_0^2 I_{M})^{-1} & 0 \\ 0 & (C_s^{(o)} + \sigma_0^2 I_{M})^{-1} \end{bmatrix}.$$  

(22)

Equation (9) simply results by substituting (22) in (20).

**APPENDIX II**

This appendix derives the CDF of summation of correlated central chi-squared random variables with four degrees of freedom based on the partial fraction expansion as

$$\prod_{i=1}^{M} \frac{1}{(1-j\lambda_i \omega)^2} \stackrel{A}{=} \sum_{i=1}^{M} \frac{A_i}{(1-j\lambda_i \omega)^2} + \sum_{i=1}^{M} \frac{B_i}{(1-j\lambda_i \omega)}$$  

(23)

where

$$A_i = \prod_{k=i}^{M} \frac{1}{(1-j\lambda_k \omega)^2} \Bigg|_{\omega = -j/\lambda_i} = \prod_{k=i}^{M} \frac{1}{(\lambda_k /\lambda_i - 1)^2}$$  

(24)

and

$$B_i = \frac{d}{d\omega} \left( \prod_{k=i}^{M} \frac{1}{(1-j\lambda_k \omega)^2} \right) \Bigg|_{\omega = -j/\lambda_i} = \left( \prod_{k=i}^{M} \frac{1}{(1-j\lambda_k \omega)^2} \right) \left( \sum_{m=i}^{M} \frac{2j\lambda_m}{1-j\lambda_m \omega} \right) \frac{1}{(\lambda_k /\lambda_i - 1)^2}.$$  

(25)

**5. CONCLUSIONS**

A novel technique for merging polarization and spatial diversities based on the motion of a doublet of crossed dipoles was proposed for handheld terminal applications. The theoretical evaluations and the experimental analysis demonstrate the effectiveness of this synthetic array combination technique for improving signal detectability in spatially correlated multipath channels. In addition, a closed
Consequently by letting $\omega = -j/\lambda$

$$B_i = \left( \prod_{k=1}^{M} \frac{1}{\lambda_k/\lambda_i - 1} \right) \left( \sum_{m=1}^{M} \frac{-2\lambda_m}{\lambda_i - \lambda_m} \right) = A \sum_{m=1}^{M} \frac{-2\lambda_m}{\lambda_i - \lambda_m}.$$  \hspace{1cm} (26)

Therefore

$$f_{SCL}(t) = \sum_{i=1}^{M} \int_{-\infty}^{\infty} \frac{1}{1 - j\lambda_i \omega} \exp(-j\omega t) d\omega$$

$$= \sum_{i=1}^{M} \int_{-\infty}^{\infty} \frac{A_i}{1 - j\lambda_i \omega} \exp(-j\omega t) d\omega$$

$$+ \sum_{i=1}^{M} \int_{-\infty}^{\infty} \frac{B_i}{1 - j\lambda_i \omega} \exp(-j\omega t) d\omega$$

$$= \sum_{i=1}^{M} \frac{A_i}{2\lambda_i^2} \exp\left(\frac{-t}{\lambda_i}\right) + \sum_{i=1}^{M} \frac{B_i}{\lambda_i} \exp\left(\frac{-t}{\lambda_i}\right).$$  \hspace{1cm} (27)

$$F_{SCL}(\gamma)$$ results by integrating $f_{SCL}(t)$ as

$$F_{SCL}(\gamma) = \sum_{i=1}^{M} (A_i + B_i) - \sum_{i=1}^{M} B_i \exp\left(\frac{-\gamma}{\lambda_i}\right)$$

$$- \sum_{i=1}^{M} A_i \exp\left(\frac{-\gamma}{\lambda_i}\right) \left(1 + \frac{\gamma}{\lambda_i}\right)$$

$$= 1 - \sum_{i=1}^{M} \left( A_i + \frac{\gamma}{\lambda_i}\right) + \sum_{i=1}^{M} B_i \exp\left(\frac{-\gamma}{\lambda_i}\right).$$  \hspace{1cm} (28)

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