Performance Evaluation of L2C Data/Pilot Combined Carrier Tracking

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ABSTRACT

Inclusion of the pilot channel in addition to the navigation data channel is considered one of the major changes in GNSS modernization. A pilot channel undergoes similar path delays and shifts as a data channel, thus making data/pilot combined tracking attractive. This combination is often not straightforward for carrier phase tracking due to the presence of data bits on the data channel. In addition, the inherent advantages of a pure PLL are often lost in such a combination. Two novel methods to combine the data/pilot channels effectively without compromising the advantages gained by using a pure PLL are proposed. These two methods are evaluated in comparison with architectures already available in literature. Results indicate that the proposed algorithms perform closer to a PLL in terms of minimum required $C/N_0$ to maintain lock with the added advantage of reduced tracking jitter.

INTRODUCTION

The current constellation of GPS is being modernized at L2 (L2-Civilian) and L5 frequencies. Apart from the usage of codes with improved correlation properties as compared to the legacy GPS L1 C/A signal, both L2C and L5 include a data-less channel (pilot) in addition to the data channel. The inclusion of a pilot channel allows a pure Phase Locked Loop (PLL) for carrier phase tracking. Normally, the phase discriminator used in a pure PLL has an extended linear region over $[-\pi, +\pi]$, thus avoiding the $\pm\pi$ ambiguity due to phase wrapping in a Costas loop. Further, a pure PLL has an improved tracking threshold of 6 dB in comparison with the Costas loop (Kaplan 2006). The absence of data bits on the pilot channel allows longer coherent integration times, an important aspect of weak signal tracking. The focus of this paper is L2C signals.

The L2C signal carries the data and pilot channels in a chip-by-chip time multiplexed fashion. The data channel carries a moderate length code (CM) of length 10,230 chips and the pilot channel carries a long length code (CL) which is of length $767250$ chips. The codes are clocked at 511.5 kHz, thus the time multiplexed code is at 1.023 MHz. A unique property of the L2C data channel is the alignment of the data bits (20 ms) with the CM code period. After CM acquisition, the data bit boundaries are known within a fraction of a chip length and this avoids the need for a separate bit synchronization algorithm. Further, synchronization between the CM and CL code reduces the search space for CL acquisition to 75 distinct possibilities.

Standard tracking architectures for L2C, often assume a zero padded local code generator as described by Tran & Hegarty (2003) to account for the time multiplexed nature of the received signal. This causes a 3 dB loss since the power in the pilot channel is neglected while tracking the data channel and vice versa. However, this avoids the cross correlation noise between the data and pilot channels. The same is assumed in this work for acquiring and tracking the L2C signal. By standard tracking architecture, a normal PLL + DLL with respective error

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discriminator, Loop Filter (LF) and a Numerically Controlled Oscillator (NCO) are referred to.

The L2C signal can be tracked as two distinct channels, namely the data and pilot for every PRN. But the data and pilot channels undergo the same path delays and shifts before reaching the receiver. Thus, they carry the same tracking information across the channels. Further, the cross correlation between the noise corrupting the coherent integration outputs on the data and pilot channel is given by the level of cross correlation between the codes on the respective channels (Van Dierendonck et al 1992). In the case of L2C, this corresponds to the cross correlation between the respective CM and CL codes, which has been shown to be in the order of  \(-40\) dB even for shorter coherent integration times of 1 ms (Muthuraman et al 2007). Zero-padded local codes further ensure nearly zero noise cross-correlation across the channels. Thus the data and pilot channel coherent integration outputs could be considered as two statistically independent observations of the same tracking parameters, namely the code phase error (\(\delta r\)), carrier phase error (\(\delta \phi\)) and the carrier Doppler (\(\delta \omega\)). This makes data/pilot combined tracking more attractive.

Different methods have been proposed in the literature to effectively make use of the availability of a pilot channel in addition to the data channel, mostly in the context of L5 signals. Spilker & Van Dierendonck (1999) suggests a non-coherent combination of the data and pilot channel coherent integration outputs for use in L5 code tracking. For carrier phase tracking, available literature suggests tracking only the pilot channel using a simple PLL, owing to the ability to track weaker signals without the \(\pm \pi\) ambiguity and due to the limitation imposed by the minimum required \(C/N_0\) to resolve for the abrupt/unknown data bit sign changes (Ries et al 2002, Macabiau et al 2003). Still, it is worthy to look at data/pilot combined tracking considering the advantage of noise reduction at nominal \(C/N_0\) levels.

One possible option, not specific to code tracking, is to combine the data and pilot channel coherent integration outputs based on the maximum power constraint (Mongrédien et al 2006). Here the data bit is accounted for using a hard decision approach, whose reliability is limited by the Bit-Error-Rate (BER) at lower \(C/N_0\). Another possible option is to employ hybrid discriminators that directly utilize the data and pilot channel coherent integration outputs to derive the carrier frequency errors (Muthuraman et al 2007). Hegarty (1999) suggests combination at the discriminator level. The data/pilot coherent integration outputs are allowed to pass through their respective discriminators and then their outputs are combined using weights. Let \(\sigma_{\text{data}}^2\) and \(\sigma_{\text{pilot}}^2\) represent the variance of the discriminator output on the data and pilot channels respectively. Then the weighting factors \((\alpha_{\text{data}}\) \& \(\alpha_{\text{pilot}}\)) are calculated as

\[
\begin{align*}
\alpha_{\text{data}} &= \frac{\sigma_{\text{pilot}}^2}{\sigma_{\text{data}}^2 + \sigma_{\text{pilot}}^2} \quad \text{&} \quad \alpha_{\text{pilot}} = \frac{\sigma_{\text{data}}^2}{\sigma_{\text{data}}^2 + \sigma_{\text{pilot}}^2} \\
\end{align*}
\]  

The combination is given by,

\[
D_{\text{out}} = \alpha_{\text{data}}D_{\text{data}} + \alpha_{\text{pilot}}D_{\text{pilot}}
\]

where, \(D_{\text{data}}\) \& \(D_{\text{pilot}}\) are the discriminator outputs on the data and pilot channels respectively. \(D_{\text{out}}\) is the discriminator output after the combination. This is then passed through the Loop Filter (LF) before feeding the error signal to the NCO on both channels. The combination is optimal in the sense that the variance of discriminator output after combination is minimized. Thus the above mentioned combination should reduce the tracking error variances by \(3\) dB under ideal conditions. The above mentioned combination is not straight forward to implement. Identical coherent integration times across the data and pilot channel is assumed to overcome the complexity. The same assumption is used herein. An alternate option to overcome this limitation is given in Tran & Hegarty (2002) and Ries et al (2002). The second limitation arises from the choice of discriminator. The most obvious option is to use a Costas discriminator on both the data and pilot channels. However, the inherent advantages of using a PLL are lost in such a formulation. If a pure PLL discriminator is used on the pilot channel and a Costas discriminator on the data channel, the critical problem to be addressed is the difference in the linear regions of the two discriminators. Costas discriminators are linear over \(\pm \pi\) whereas PLL discriminators are linear over \(\pm \pi\). Methods to detect and/or compensate for such jumps due to dynamics or noise at lower \(C/N_0\) can be found in Ries et al (2002), Tran & Hegarty (2002) and Julien (2005). Here again the performance depends on the reliability of the decision algorithms at lower \(C/N_0\) as mentioned above. Further, the noise variance of arc-tangent discriminators (ATAN – Costas, ATAN2- PLL) considerably deviate from the theoretical values at lower \(C/N_0\), thus rendering any “pre-calculated” weights to be invalid (Julien 2005). In this work, the weights for the discriminator combination are calculated by running “on-the-fly” variance estimators on the data and pilot channel discriminator outputs (Moir 2001) and by imposing the normalizing constraint

\[
\sum_n \alpha_n = 1 
\]

The loop parameters for the variance estimation are chosen based on the scenario at hand.
For the combination of code discriminators, phase jumps due to data bit sign change is not a problem as most discriminators utilize the early-late power difference.

**MOTIVATION**

This work evaluates the performance of different data/pilot combined carrier phase tracking algorithms along with the following two algorithms that are proposed for the same purpose:

(i) Utilizing PLL discriminators on both channels

(ii) Kalman filter based data/pilot combined tracking

To make a fair comparison between the standard tracking architectures and Kalman filter based tracking, a novel noise bandwidth tuning algorithm is also proposed. The role of $C/N_0$ estimators in Kalman Filter based tracking and adaptive bandwidth tracking is also briefly discussed in the following sections.

**PLL DISCRIMINATOR ON DATA CHANNEL**

The arc-tangent discriminators ATAN (Costas) and ATAN2 (PLL) provides unity gain over the region corresponding to phase errors of $\pm \frac{\pi}{2}$ and $\pm \pi$ respectively. But as $C/N_0$ decreases, the linear operating region gradually starts to degrade around their respective boundaries. When an ATAN discriminator is used on the data channel and an ATAN2 discriminator on the pilot channel, for phase errors (original or $\pi$-wrapped) around the boundary of the ATAN discriminator’s linear region, the estimates on the data channel become biased for lower $C/N_0$, as compared to using an ATAN2 discriminator on the data channel. Thus, utilizing an ATAN2 discriminator on the data channel as well, can help to avoid such biases. This effect was simulated and is illustrated in Figure 1 & Figure 2. In Figure 2, for phase errors close to the boundary of the ATAN linearity region (ex. $\delta \phi = 80^0$), significant degradation in bias could be observed as $C/N_0$ decreases. In Figure 1 however, for phase errors close to the origin (ex. $\delta \phi = 10^0$) no considerable difference in performance as a function of $C/N_0$ could be observed. The dotted lines correspond to the $\pm 1 \sigma$ noise variance in the phase estimates. The variation in noise performance across the choice of two discriminators remains the same.

**KALMAN FILTER BASED DATA/PILOT COMBINED TRACKING**

With the growing interest in signal post-processing, where there is no constraint on processing time, Kalman Filter based tracking is gaining importance in software receivers (Psiaki & Jung 2002, Petovello & Lachapelle 2006, Mongredien et al 2007). A Kalman filter for tracking is normally modeled to accept the coherent integration outputs of the early, prompt and late channel. In this work, the state space and observation model as described in Petovello & Lachapelle (2006) are made use of. Here again it is possible to design two independent instances of KF based algorithms to track the data and pilot channels for every PRN. It is also possible to combine the data and pilot channel coherent integration outputs before feeding the result as observations to the Kalman filter. In that case, the observation model does not change as compared to that of L1 C/A. This was demonstrated for L5 signals by Mongredien et al (2007). A similar approach could be found in Ziedan (2005) for L2C as well as L5 signals. The main difference being the former utilizes a hard decision based (or) maximum power combining approach whereas the latter uses Maximum Likelihood (ML) estimation of possible bit combinations across the coherent integration period. This curtails the freedom of independently weighting the channels based on their reliability. In this work, the data and pilot channel coherent integration outputs are directly passed on to the Kalman Filter as

![Figure 1](image1.png)

*Figure 1: Mean deviation from original phase error for ATAN and ATAN2 discriminators on the data channel for $\delta \phi = 10\degree (\delta f = 0, T_{coh} = 20\ ms)$*

![Figure 2](image2.png)

*Figure 2: Mean deviation from original phase error for ATAN and ATAN2 discriminators on the data channel for $\delta \phi = 80\degree (\delta f = 0, T_{coh} = 20\ ms)$*
independent observations of the states to be estimated. Although this increases the computational complexity, this is carried on so as to provide a reference to compare the performance of different standard tracking architectures for channel combination.

ADAPTIVE BANDWIDTH FOR STANDARD TRACKING

To enable fair comparison of standard tracking architectures with the KF based data/pilot combination, this section presents a simple, yet novel way to tune the noise bandwidth \( B_n \) of the Loop Filter (LF). For simplicity, a second order loop (PLL) is taken for study. There are two primary design approaches for tuning the bandwidth. One is to aid the PLL in frequency acquisition (Kim et al 2003) and the other is to reduce the steady state error. The latter is of concern in this work. A second order loop is sensitive to acceleration stress [Kaplan 2006] and the thermal noise \([\text{Kim et al 2003}]\) due to acceleration stress on a second order PLL is given by

\[
\sigma_{PLL,t} = \sqrt{\frac{B_n}{(C/N_0)_r}} \left( 1 + \frac{1}{2T_{coh}(C/N_0)_r} \right) \text{ rad} \quad (4)
\]

where \( B_n \) is the noise bandwidth in units of Hz, \((C/N_0)_r\) is the \( C/N_0 \) expressed as a ratio in Hz, and \( T_{coh} \) is the coherent integration time in seconds. Similarly, the steady state error \( \left( \theta_e \right) \) due to acceleration stress on a second order PLL is given by

\[
\theta_e = \frac{\delta a}{\omega_n^2} \text{ rad} \quad (5)
\]

where \( \delta a \) is the acceleration error in \text{rad/s}^2 and \( \omega_n \) is the undamped natural frequency of the second order system under consideration in \text{rad}. Ignoring the other sources of phase jitter, the goal is to choose a \( B_n \) that minimizes the net error given by \( \sigma_{net} \)

\[
\min_{B_n} \sigma_{net} = \min_{B_n} \left( \sigma_{PLL,t} + \frac{\theta_e}{3} \right) \quad (6)
\]

Loop stability is imposed as a constraint on the minimization. The transfer function \( H(s) \) of the second order PLL is given by

\[
H(s) = \frac{2\eta\omega_n s + \omega_n^2}{s^2 + 2\eta\omega_n s + \omega_n^2} \quad (7)
\]

where \( \eta \) is the damping factor and \( \omega_n \) is the un-damped natural frequency. Routh’s criterion for stability demands \( 2\eta\omega_n > 0 \) & \( \omega_n^2 > 0 \). Assuming \( \eta = 0.707 \), which is suitable for most tracking applications, the two conditions above simplify to one, \( \omega_n > 0 \). This could also be interpreted as, if \( \omega_n > 0 \) is met, then the poles of the characteristic equation \( -\eta\omega_n \pm j\omega_n\sqrt{1-\eta^2} \) will lie in the left half of the s-plane, thus ensuring stability. For a given \( \eta \) and \( \omega_n \), the noise bandwidth \( B_n \) is given by [Gardner 2005]

\[
B_n = \frac{\omega_n}{2} \left( \eta + \frac{1}{4\eta} \right) = \gamma \omega_n \quad (8)
\]

where \( \gamma = 0.5303 \) for \( \eta = 0.707 \). Thus the stability criterion maps to non-zero noise bandwidth \((B_n > 0)\).

On minimizing, the first derivative test gives,

\[
\bar{B}_n = \sqrt{\frac{16(\delta a)^2 \gamma^4}{9\beta}} \quad (9)
\]

where \( \bar{B}_n \) is the noise bandwidth estimate and \( \beta = \frac{1}{(C/N_0)_r} \left( 1 + \frac{1}{2T_{coh}(C/N_0)_r} \right) \). A positive estimate of \( \bar{B}_n \) ensures the second derivative test for minima is met (since \( \beta, \delta a, \gamma > 0 \)).

When \( \frac{C}{N_0} \to \infty \), \( \beta \to 0 \) which makes \( \bar{B}_n \to \infty \), which is expected. But for the other extreme condition, as \( \delta a \to 0 \), \( \bar{B}_n \to 0 \). This violates the stability criterion. One way to overcome this is by setting a hard minimum value on the allowed range of the noise bandwidth \((\bar{B}_n \geq 0.05 \text{ Hz})\).

In the previous formulation for the noise bandwidth estimate, the acceleration was assumed to be a known constant. One simple way to estimate the acceleration (\( \delta a \)) when the PLL is correcting for the phase errors is to look at a filtered version of the phase discriminator output. This method exploits the steady state error from (5). The phase discriminator outputs are passed through a 2nd order Butterworth filter with a tight bandwidth of 0.5 Hz to extract the dc component, which is the steady state phase error. This filter will be referred as the “steady-state” filter to distinguish it from the loop filter of the PLL. Let the phase discriminator outputs be given by \( x_k \), which is fed to the steady state filter. Then the output \( y_k \) is given by

\[
y_k = \sum_{n=0}^{2} c_n x_{k-n} + \sum_{n=1}^{2} d_n y_{k-n} \quad (10)
\]

where \( c_n \) & \( d_n \) are the weighting constants. The acceleration estimate \( \delta a \) is calculated from the steady state filter output using (5). Assume, the noise bandwidth of the LF is not changed over \( \{x_k, x_{k-1}, x_{k-2}\} \). Then the corresponding acceleration estimate \( \delta a \) is given by \( y_k \times \omega_n^2 k \), where \( \omega_n k \) corresponds to the un-damped natural frequency at time \( k \). Based on this, the new noise bandwidth is calculated using (9), and hence the new value for \( \omega_n(k+1) \) will be updated using (8). This is reflected in the new steady state error at time \( k + 1 \). So
care must be taken that \( \{x_k, x_{k-1}, y_k, y_{k-1}\} \) are scaled appropriately before calculating \( y_{k+1} \). The scaling factor \( SF \) is given by

\[
SF_k = \frac{\alpha^2_{1,n,k-1}}{\alpha^2_{1,n,k}} \tag{11}
\]

This method to adapt bandwidth is initially validated using simulations and then with live signals currently being transmitted by the IIR-M satellites. Figure 3 shows the Doppler plot obtained for PRN 31 at \( \approx 32 \text{ dB } - \text{Hz} \) over a period of 120 s along with a linear fit for the same. From the linear fit, a rough estimate of the acceleration could be made \( \approx -0.17 \text{ Hz/s}^2 \). Using this information, (9) yields a noise bandwidth estimate of \( \approx 3 \text{ Hz} \). Figure 4 shows the time series of noise bandwidth estimates produced by the adaptive noise bandwidth algorithm proposed, which is in-line with the expected value from the linear fit. The loop bandwidth is updated every \( T_{coh} \) and the \( C/N_0 \) estimates are fed from the \( C/N_0 \) estimator. A FLL is used to acquire frequency lock over the first 1 s, resulting in the constant noise bandwidth of 10 Hz (external initialization) over that period of time for the PLL.

As observed from the (9), the quality of the \( \hat{B}_n \) estimate is affected by the noise levels in the acceleration estimate as well as the \( C/N_0 \) estimators.

**DIFFERENTIAL C/N₀ ESTIMATORS**

In a Kalman filter based tracking algorithm, the noise variance of the observations are modeled based on the \( C/N_0 \) estimators. Similarly, in standard tracking architectures, using the adaptive bandwidth tuning algorithm as proposed, also relies on the quality of the \( C/N_0 \) estimators being used. A standard \( C/N_0 \) estimator relies on computing the ratio of the Narrow Band Power (NBP) to the Wide Band Power (WBP) (Van Dierendonck 1995). Other options include using a differential estimator. The formulation provided here is similar to [Pany & Eisfeller 2006]. Let the coherent integration output be given by

\[
Y(t_1 + T_{coh}) = N A d_k \text{sinc}(δf T_{coh}) \exp \left( j[δφ + δω(t_1 + T_{coh})] \right) + n_c \tag{12}
\]

where \( Y(t) \) is the complex form of the coherent integration output at time \( t \), \( N \) is the number of samples accumulated in the integration period, \( A \) is the amplitude, \( d_k \) is the data bit over the integration period from \( t_1 \rightarrow t_1 + T_{coh} \), \( δf \) and \( δω \) are the frequency errors expressed in Hz and rad/s respectively, \( δφ \) is the phase error in rad and \( n_c \) is the complex noise sample. \( n_c \). The complex noise sample can be expressed as,

\[
n_c = n_i + j n_q \tag{13}
\]

where \( n_i \) and \( n_q \) represent the in-phase and quadrature noise respectively. The variance of the noise is given by \( n_c \sim N(0, \sigma_n^2) \) where \( \sigma_n^2 \) corresponds to the variance of the noise sample corrupting the digitized RF signal.

The signal power could be estimated from the coherent integration outputs as,

\[
E[\Re(Y(t_1 + T_{coh})Y^*(t_1))] = E[\Re(A^2 N^2 d_k d_{k-1} \text{sinc}^2(δf T_{coh}) \exp(j[δωT_{coh}]))] \tag{14}
\]

where \( d_k \) corresponds to the data bit in the period \( t_k - T_{coh} \rightarrow t_k \) and \( \Re(.) \) denotes the real part of the complex number. Equation (14) is valid under the following assumptions

(a) The signal and noise are independent i.e.,

\[
E[A n_i] = 0 \forall t
\]

(b) Zero cross correlation between the noise samples across coherent integration outputs i.e.,

\[
E[n_c(t + T_{coh})n^*_c(t)] = 0 \forall t
\]

**Figure 4**: Noise bandwidth estimates obtained using adaptive noise bandwidth algorithm for PRN 31.

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**Figure 3**: Doppler plot for PRN 31 for independent data and pilot channel tracking, with a linear fit for the same.
The phase error in it. Residual frequency or frequency rate errors might considerably degrade a differential $C/N_0$ estimator as the length of averaging time increases due to the residual phase error in it.

Reliability of different $C/N_0$ estimators is quantified using the Spirent GSS7700 hardware simulator and shown in Figure 5. The simulator is configured to generate a constant $\approx 32 \, dB - Hz$ signal for the first 35 s and this is used as a reference. Later, at known time instants, the generated signal power is reduced in known steps (as given by the x-axis in Figure 5). The simulator is allowed to stay at each step for 30-45 s to allow statistical inference. The standard $C/N_0$ estimator which uses 20 ms of coherent integration on the pilot channel ($1 ms \times 20$ coherent summations) to estimate the narrow band signal power is given for reference (since this is the most commonly employed in receivers). Other estimators under consideration include a standard estimator with a 100 ms net coherent output for NBP again on pilot channel, and differential & hybrid estimators using 20 ms coherent outputs (averaging over 50 outputs). Care is taken that, for hybrid estimators, a data/pilot combined carrier tracking was in place to ensure equal phase/frequency errors across the channels, while other estimators were in an independent channel tracking mode. The mean and 1σ standard deviation of the attenuation, as obtained from the estimators, are averaged across 6 PRN’s. As observed from Figure 5, beyond 9 dB ($\approx 23 \, dB - Hz$) of attenuation, the reference $20 \, ms - net$ standard $C/N_0$ estimator starts to fail considerably in terms of the bias in estimate. Similarly, a differential estimator running on the data channel provides reliable estimates for up to 13.5 dB of attenuation ($\approx 18.5 \, dB - Hz$). Beyond that, the data channel loses lock and hence the bars do not appear in Figure 5 for attenuations $\geq 15 \, dB$. All other three estimators are able to provide ‘nearly’ unbiased estimates of the attenuation until 18 dB of attenuation ($\approx 14 \, dB - Hz$) beyond which the carrier tracking lost lock on both channels. Either a standard estimator with $20 \, ms \times 5$ coherent summations or a differential estimator using $20 \, ms$ (both on pilot channels alone) is considered as a potential candidate. The latter is used in this work. Although hybrid $C/N_0$ estimators are able to provide unbiased estimates to the same extent, they are not considered further in this work due to the increase in noise levels arising out of missed/false data bit sign detections at lower $C/N_0$ (approximately $14 \, dB - Hz$).
TEST METHODOLOGY

The proposed algorithms were validated using live L2C signals and/or data collected using the Spirent GPS simulator, namely the GSS7700. The down-converted and digitized (8-bit) data was collected using a National Instruments RF front-end at a sampling rate of 2.5 MHz. This collected data was then post-processed using a custom modified version of the PLAN group’s GSNRx™ software. In the software receiver, a FLL is initially used to acquire frequency lock after acquisition. After acquiring frequency lock, carrier phase tracking using PLL/Costas was turned on. The bandwidth of the FLL and DLL was maintained constant across the implementations. The carrier phase tracking noise bandwidth was tuned adaptively using the algorithm mentioned previously. The hard minimum value on the noise bandwidth tuning algorithm was set to 0.05 Hz. For Kalman filter based tracking, after achieving frequency lock, a PLL was used to close the loop before making a handover to the KF based tracking loops. Results were obtained at the steady state after allowing sufficient time for all the above initializations.

RESULTS

For analysis, the simulator was configured to generate a fixed “received power” signal across the satellites and this signal power was gradually reduced in known steps (~1.5 dB) at known intervals (30 s). The receiver was allowed to track the data collected with different carrier phase tracking architectures, which include

(a) Independent data channel tracking with a Costas discriminator (Sign(I) × Q)
(b) Independent pilot channel tracking with a pure PLL (ATAN2)
(c) Data-pilot combined tracking with Costas (ATAN) discriminators on both channels
(d) Data-pilot combined tracking with ATAN2 on both channels
(e) Data-pilot combined tracking with a Kalman Filter

The above said implementations were compared based on (i) minimum required $C/N_0$ before losing carrier frequency lock (ii) tracking jitter. The former was quantified by declaring loss of lock on a channel in a given time interval if the Doppler tracking is observed to diverge in that period. The latter was quantified by passing the Doppler frequency residuals through a filter with a transfer function $H_{NCO}(Z)$, equivalent to the Numerically Controlled Oscillator (NCO), given by

$$H_{NCO}(Z) = \frac{Z^{-1}}{1 - Z^{-1}} \quad (19)$$

and measuring the noise variance at the output of the filter. Residuals of Doppler frequency were obtained by removing the deterministic component (Doppler frequency + Doppler frequency rate) in the Loop Filter (LF) output with a first order fit over every 200 samples. To ensure fair comparison, the tracking loop was updated at every code period, corresponding to a $T_{coh}$ of 20 ms, across all the architectures described. The Kalman filter based tracking results are not presented in the tracking jitter plot due to their dependence on the user’s choice of system noise values.

Figure 6 shows the minimum $C/N_0$ over which the channel was able to maintain Doppler frequency lock. Figure 7 shows the tracking jitter across the different architectures with reference to a pure PLL on the pilot channel. The results presented are averaged across 6 PRN’s. As expected, the PLL on the pilot channel is observed to maintain lock over higher attenuations ($\approx 15 \text{ dB} - \text{Hz}$) than the others.

The “data channel only” tracking loses lock much earlier than the others, due to the error in resolving data bit signs. A difference of about $\approx 4 \text{ dB}$ is observed between the independent data and pilot channel tracking. In terms of tracking jitter, the performance is closer to a pure PLL (within ~ 1 dB) over the region where it is able to maintain lock.

Using a Costas discriminator on both channels (data & pilot) and combining them aids in reducing the minimum required $C/N_0$ by 1 dB in comparison with the data channel only tracking. Still the advantage largely comes out of the reduction in tracking jitter ($\approx 3 \text{ dB}$).

Using ATAN2 on both channels, initially provides the same gain in terms of reduction in noise variance as compared to the Costas combination, but this gain is observed to decrease to around $1.5 \text{ dB}$ at lower $C/N_0$, owing to the inability to correct for the abrupt phase jumps under such attenuated conditions. Still, it is able to maintain lock over the same level of attenuations as a

![Figure 6: Comparison of different implementations based on minimum required $C/N_0$ before losing lock on the channel](image-url)
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CONCLUSIONS

Data/pilot combined tracking using Costas discriminators on both channels serves little in terms of reducing the minimum required $C/N_0$ to maintain lock as compared to a data channel only tracking. However, combined tracking using Costas discriminators on both channels does prove to be useful for reducing the noise variance over the region where it is able to maintain lock. In contrast, when PLL discriminators are used on both channels, though the gain in noise reduction is less compared to the Costas combination for lower $C/N_0$, the PLL discriminators do aid in maintaining lock to the same level of attenuations as compared to a pure PLL. Similar results were obtained using the Kalman filter based data/pilot combination. Thus the proposed two combinations effectively make use of the statistically independent information across the channels even at lower $C/N_0$ without losing lock. The analysis is limited by the fact that minimum required $C/N_0$ is defined based on the architecture’s ability to maintain Doppler frequency lock over a period of 30 s for every attenuation. Theory to establish the exact values of tracking jitter across the different architectures is currently under development.

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Figure 7: Comparison of NCO tracking jitter across different implementations

pure PLL on pilot channel. Naturally, maintaining lock is much more important than reducing the tracking jitter.

The data/pilot combination with KF based tracking provides almost the same performance as a pure PLL on the pilot channel. Here it should be mentioned, that the performances obtained for KF based tracking algorithms are for one set of noise parameters. Ability to maintain lock with reduced $C/N_0$ is found to be largely a function of the parameters chosen for the system noise model (code-carrier divergence, acceleration noise, clock model etc.). The same is applicable for the tracking jitter observed from the KF based estimates.


