Combined Spatial-Polarization Correlation Function for Indoor Multipath Environments

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Abstract—The combined spatial-polarization correlation formulation for a multipath environment based on the uniform sphere of scatterers model is developed. The closed-form expressions developed are useful for quantifying the potential diversity gain in antenna diversity systems. An extensive set of measurements were carried out at 1947.5 MHz to verify the theoretical results. The proposed model can be utilized for characterizing spatial and polarization diversity systems for indoor multipath environments.

Index Terms—Antenna diversity, correlation, diffuse multipath, polarization, spatial.

I. INTRODUCTION

Spatial diversity techniques have proven to be effective in mitigating multipath fading and have therefore been integrated into wireless communication systems. Received signals on the various diversity branches decorrelate spatially with the rate of decorrelation increasing monotonically with the multipath angular spread [1]. Likewise, polarization diversity based on antennas of different polarization provides an additional means of attaining diversity [2]–[4]. Here, the correlation between signals of two similar colocated antennas decreases by increasing the separation of the antennas’ polarization vectors with orthogonal polarizations, resulting in approximately uncorrelated diversity branches [2], [3]. Combined spatial movement and polarization rotation results in an overall lower branch correlation and therefore the potential of higher diversity gain as will be shown via the closed-form expressions generated.

Degradations of the diversity gain are attributable to nonoptimal branch power ratios, system losses, and correlation between the diversity branches. Branch correlation depends entirely on the underlying model representing the multipath scattering. Typically stochastic multipath models assume jointly Gaussian processes such that correlation of represented diversity branches can be considered pairwise. Clark’s model, based on a dense ring of scatterers in the azimuth with the receiving antenna near the center of the ring, results in a practical expression for the signal correlation as a function of spatial separation of the antennas [5]. A modification to Clark’s model was discussed by [6] based on a sphere of scatterers, and it was argued to be more suitable for indoor multipath environments since enclosed room cavities are more akin to an approximating sphere than an azimuthal ring of scatterers.

A theoretical model for characterizing correlation between diversity branches in a polarization diversity system at a base station was first established by Kozono et al. [7], who assume a narrow multipath beam. Vaughan [8] further extended Kozono’s model to account for the rotation of two antennas with a fixed angular separation around their phase center. However, both Kozono and Vaughan models assume an incoming multipath beam with narrow azimuth spread such that their results are more applicable to base-station diversity systems. Correlation between orthogonal components of the electromagnetic field at the mobile was discussed by [9], which is based on a uniform but arbitrarily wide angular spread. More recently, Brown [10] has established a model to account for an arbitrary distribution of multipath power in azimuth and elevation at the mobile based on orthogonal horizontal and vertical receive polarizations. However, the idealized spherical distribution of scatterers provides three orthogonal diversity branches at the receiver. Three polarizations were considered by [11], however with a scattering model based on a ring of scatterers, which is less applicable to indoor environments than the sphere of scatterers.

With the increasing interest in utilizing joint spatial and polarization diversity at the mobile, e.g., [12] and [13], quantifying the combined spatial-polarization diversity at the mobile is necessary. Although generalized formulation for evaluating antenna correlation is given in [8], there are few multipath models that result in practical formulations and, to the extent of the authors’ knowledge, are discussed either for spatial diversity or polarization diversity systems with the exception of [14], which proposes an empirical equation based on measurements in a reverberation chamber.

Therefore, the impetus of this letter is the development of an expression of spatial-polarization diversity based on the spherical scattering model. This is based on determining expressions for the combined spatial-polarization correlation function based on an isotropic antenna. Although helpful in simplifying the equations, a linearly polarized antenna with an isotropic pattern does not exist. Consequently, evaluating polarization diversity in the context of the isotropic radiator is unrealistic. Therefore, this letter extends the formulation into a more realistic antenna structure and deduces closed-form expressions for the combined correlation function. Nevertheless, and in order to provide comparison to previous works, the correlation formulation based on an isotropic antenna is also given here.
The rest of the letter is organized as follows. Section II defines the system model and basic assumptions, Section III derives the spatial-polarization correlation function, Section IV presents the measurement results, and Section V presents the conclusion.

II. SYSTEM MODEL AND ASSUMPTIONS

A uniform sphere of scatterers is assumed as it is suitable for characterizing indoor multipath as well as yielding simple closed-form expressions for the spatial-polarization correlation [6], [12]. Following Collin and Zucker [15], the open-circuit voltage $V$, induced at an antenna output, can be found as

$$V = \int \mathbf{E}_0(\Omega) \cdot \mathbf{E}_0(\Omega)d\Omega$$  \hspace{1cm} (1)

where $\Omega$ is the solid angle $(\theta, \varphi)$, $\mathbf{E}_0(\Omega)$ is the incident electric field from sources arriving from a solid angle $d\Omega$ at the receiving antenna, and $\mathbf{E}_0(\Omega) = \mathbf{E}_0(\theta, \varphi) = E_0 \hat{\theta} + E_\varphi \hat{\varphi}$ is the antenna’s normalized far-zone radiated electric field, which is also known as the vector effective length (VEL) as defined in [16]. The unit vectors of $\hat{\theta}$ and $\hat{\varphi}$ are defined as the elevation and azimuthal directed vectors in a right-handed spherical coordinate system.

Source polarizations $E_{\theta}, E_{\varphi}$ are assumed to be statistically spatially white and identically distributed. Also, according to the spherical scattering model, the sources are assumed to be uniformly distributed on the spherical surface in the far field of the receiving antenna. Consequently, it can be shown that $V$ is a zero-mean complex normal random variable $\text{CN}(0, \sigma^2)$, where $\sigma^2$ is the variance and is related to the received signal power at the mobile antenna [17].

The goal is to evaluate the correlation $\rho(p, \psi)$ between the open-circuit voltages of two identical antennas I and II with spatial-polarization separation of $(p, \psi)$, where $p = |\mathbf{P}_1 - \mathbf{P}_\Pi|$, $\mathbf{P}_1, \mathbf{P}_\Pi$ are the position vectors of the antennas, and $\psi$ is the angular separation between antennas’ polarization vectors as illustrated in Fig. 1. The pair of antennas is assumed to be ideal in that they are perfectly linearly polarized and have no mutual coupling. As a result, the deduced analytical expressions predict signal decorrelation arising from spatial-polarization translation of a single antenna and should therefore be utilized with caution when multiple-antenna arrays with closely spaced elements are being studied.

Due to the spherical symmetry of the sources, and without any loss of generality, we assume that both antennas are located on the $xz$ plane of the Cartesian coordinate system. Consequently

$$\rho(p, \psi) = \langle V_1 V_\Pi^* \rangle / \sqrt{\langle V_1^2 \rangle \langle V_\Pi^2 \rangle}$$  \hspace{1cm} (2)

where $^*$ is Hermitian, and $(\cdots)$ is an expectation operation with respect to the random properties associated with the distributed scatterers. Consequently and following [8]

$$\langle V_1 V_\Pi^* \rangle = \left\langle \int \mathbf{E}_0(\Omega_1) \cdot d\Omega_1 \int \mathbf{E}_0^*(\Omega_2) \cdot d\Omega_2 \right\rangle.$$  \hspace{1cm} (3)

The expectation operator and the angular integrals are interchangeable as there are no singularities involved. Furthermore, considering that $\mathbf{E}_0$ is spatially white such that $\langle E_{\theta}(\Omega_1) E_{\theta}(\Omega_2) \rangle = \langle E_{\varphi}(\Omega_1) E_{\varphi}(\Omega_2) \rangle = c \delta(\Omega_1 - \Omega_2)$, where $c$ is a normalizing scaling constant, and $\delta(\cdot)$ is the Delta function, (3) can be written as

$$\langle V_1 V_\Pi^* \rangle = c \int \int [E_{\theta} E_{\theta}^* + E_{\varphi} E_{\varphi}^*] \delta(\Omega_1 - \Omega_2) d\Omega_1 d\Omega_2 = c \int [E_{\theta} E_{\theta}^* + E_{\varphi} E_{\varphi}^*] d\Omega_1 \int d\Omega_2 = c \int_0^{2\pi} \int_0^{\pi} [E_{\theta} E_{\theta}^* + E_{\varphi} E_{\varphi}^*] \sin \theta d\theta d\varphi.$$  \hspace{1cm} (4)

III. SPATIAL-POLARIZATION CORRELATION

A. ISOTROPIC ANTENNA

Consider a linearly polarized antenna with an isotropic pattern and a VEL of

$$\mathbf{E}_1 = \hat{\theta} e^{jk \mathbf{P}_1}$$  \hspace{1cm} (5)

where $k = -k(\hat{x} \sin \theta \cos \varphi + \hat{y} \sin \theta \sin \varphi + \hat{z} \cos \theta)$ is the propagation vector also known as the vector wavenumber with $k = 2\pi/\lambda$ and $\lambda$ the carrier wavelength, and $\mathbf{P}_1$ is the position vector.

Now consider antenna II with the position vector $\mathbf{P}_\Pi$ and the angular separation of $\psi$ with respect to antenna I as shown in Fig. 1. Consequently, the VEL of antenna II can be shown to be

$$\mathbf{E}_\Pi = (\hat{\theta} \cos \psi + \hat{\varphi} F(\theta, \varphi)) e^{jk \mathbf{P}_\Pi}$$  \hspace{1cm} (6)

where $F(\theta, \varphi)$ is a nuisance function and will be eliminated throughout the process. By replacing (5) and (6) in (2) and taking (4) into account, one obtains

$$\rho(p, \psi) = \cos \psi \frac{1}{4\pi} \int_0^{2\pi} \int_0^{\pi} e^{jk \Delta \mathbf{P} \sin \theta d\theta d\varphi}$$

$$= \cos \psi \frac{1}{4\pi} \int_0^{2\pi} \int_0^{\pi} e^{jk \Delta \mathbf{P} \sin \theta \sin \theta \sin \varphi \cos \varphi + \cos \theta \cos \theta \sin \theta \sin \varphi \cos \varphi} \sin \theta d\theta d\varphi$$  \hspace{1cm} (7)

where

$$\Delta \mathbf{P} = \mathbf{P}_1 - \mathbf{P}_\Pi = p(\hat{x} \sin \theta \cos \varphi + \hat{y} \sin \theta \sin \varphi + \hat{z} \cos \theta).$$

Due to the spherical symmetry of the scatterers, the result cannot be a function of $\theta, \varphi$. Therefore, we let $\theta = 0$, and consequently

$$\rho(p, \psi) = \sin(kp) \cos \psi.$$  \hspace{1cm} (8)
As can be seen from (8), the combined spatial-polarization correlation function is a separable function of space $p$ and orientation $\Psi$ variables.

**B. Infinitesimal Dipole**

Assume a $z$-oriented infinitesimal dipole with the following VEL:

$$
E_t = \hat{\theta}e^{jkpn} \sin \theta.
$$

(9)

The VEL of an infinitesimal dipole located at $P_\Pi$ on the $xz$ plane and oriented such that it makes an angle of $\Psi$ with the $z$-axis (Fig. 1) is

$$
E_\Pi = e^{jkpn_\Psi} \left( \hat{\theta}(j \cos \theta \cos \varphi \sin \Psi + \sin \theta \cos \Psi) - \hat{\varphi}(j \sin \varphi \sin \Psi) \right).
$$

(10)

By replacing (9) and (10) in (2) and taking (4) into account, one obtains

$$
\rho(p, \Psi) = \frac{j \sin \Psi}{\pi^2} \int_0^{2\pi} \int_0^\pi e^{jk \Delta P(\cos \theta \cos \varphi) \sin \theta d\theta d\varphi}
$$

$$
+ \frac{\cos \Psi}{\pi^2} \int_0^{2\pi} \int_0^\pi e^{jk \Delta P(\sin \theta) \sin \theta d\theta d\varphi.}
$$

(11)

Consequently, and by taking into account that the first integral at the right side of (11) is zero, one obtains

$$
\rho(p, \Psi) = \frac{\cos \Psi}{\pi} \left( \int_0^\pi e^{jkp \cos \theta d\theta} - \int_0^\pi \cos(2\theta)e^{jkp \cos \theta d\theta} \right)
$$

$$
= (J_0(kp) + J_2(kp)) \cos \Psi
$$

(12)

where $J_n$ is the Bessel function of the first kind of order “$n$.” Note that the relative spatial decorrelation for a translated antenna of fixed orientation is independent of the polarization of an incident plane wave. Likewise, the mapping of the polarization of individual sources on the far-field scattering sphere at the receiving antenna is independent of relative spatial translation of the receive antenna. Therefore, the above analysis can be extended to antennas with more complex geometries by utilizing the superposition principle. Assuming the sphere of scatterers’ model, it can be shown that for antenna geometries with collinear infinitesimal current elements such as the thin wire dipole, the combined spatial-polarization correlation function is a separable function of space and orientation separation as

$$
\rho(p, \Psi) = f(kp) \cos \Psi
$$

(13)

where $f(kp)$ represents the spatial part of the correlation function and varies for different antenna geometries. While (13) is not directly applicable for general antennas with noncollinear infinitesimal current elements, it is applicable to individual subsets of orthogonally oriented current elements.

**IV. EXPERIMENTAL RESULTS**

An extensive set of measurements was conducted to validate the analytical results of the previous section. The measurement site was a typical one-story laboratory with a variety of equipment and was verified to typify a Rayleigh fading channel [12], [19]. The experimental setup and measurement equipment are shown in Fig. 2.

The pilot CDMA signal transmitted from a nearby base station at 1947.5 MHz was captured by an active receiving half-wavelength dipole antenna. The antenna was then connected to an RF front end for down-conversion and sampling at 10 MHz. In addition, the antenna was mounted on a three-wavelength-long nonmetallic rotary arm that was positioned by a stepper motor. The rotary arm was set to rotate the dipole antenna around its phase center with a constant angular speed of $\omega = \pi/2$ rad/s and $v = 5$ mm/s.

![Measurement setup](image)

**Fig. 2. Measurement setup.**

**Fig. 3. Measured and theoretical [based on (12)] combined correlation for $\omega = \pi/2$ rad/s and $v = 5$ mm/s.**

The output signal of the rotating-moving antenna was integrated over 10-ms periods while the antenna was being translated over an equivalent length of 22 mm. Subsequently, (2) was applied to the output signal of the rotating-moving antenna to compute the magnitude of complex correlation coefficient as a function of time $t$ (see Fig. 3), which maps into the antenna’s spatial and angular displacements based on $\Psi(t) = \omega t$ (rad) and $p(t) = vt$ (m). Note that the output signal of the same antenna integrated over different positions and orientations was used to calculate the correlation, thus avoiding issues of mutual coupling between measurement antennas. A static reference antenna, located several wavelengths away from the rotating-moving antenna, was used to compensate for the residual oscillator offset and instabilities.

A potential limitation of these measurements is the issue of attaining statistical significance in the context of estimating the correlation parameters of the signal. Experimental results in this letter are based on 280 uncorrelated spatial samples per antenna orientation, which appears to be adequate based on the results. As can be seen, the experimental results are in good agreement.
with the theoretical predictions of (12). Also, note that the measured decorrelation based on the antenna rotation is not as high as expected due to the scattering objects in the near-field of the experimental antenna used. Near-field scattering violates the assumption of a sphere of scatterers in the far field. Also, the residual cross-polarization response of the experimental dipole antenna used is a factor.

V. CONCLUSION

An analytical model based on a sphere of scatterers was developed for characterizing the combined spatial-polarization correlation function of a linearly polarized antenna in an indoor environment. Two different antenna scenarios based on an isotropic antenna as well as a more realistic case of an infinitesimal dipole were considered for the analysis. The analytical model resulted in simple closed-form expressions for the combined spatial-polarization correlation function. It is evident from the analytical results that, for antenna geometries with colinear current elements, the correlation function is a separable function of space and polarization separation. Finally, an extensive set of measurements was performed in order to validate the proposed theoretical model, and the experimental results were found to be in good agreement with the theoretical predictions.

REFERENCES