GNSS Multipath Error Reduction in Harsh Environments

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BIOGRAPHY

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Professor Gérard Lachapelle holds a Canada Research Chair in Wireless Location in the Department of Geomatics Engineering, the University of Calgary, where he has been a professor since 1988 and heads the PLAN Group. He has been involved in a multitude of Global Navigation Satellite Systems (GNSS) R&D projects since 1980, ranging from RTK positioning to indoor location and GNSS signal processing enhancements.

ABSTRACT

The performance of code delay and carrier phase estimation of a GNSS signal propagated in an urban or indoor environment is highly degraded by multipath when the conventional multipath mitigation techniques such as narrow correlator and double delta correlator are applied. These conventional techniques rely on the value of the delay corresponding to the peak of the autocorrelation function.

In this paper, a modified iterative Projection Onto Convex Set (POCS) algorithm is discussed that estimates the Channel Impulse Response (CIR) using an adaptive threshold to remove the spurious noise peaks at each iteration. Herein, an adaptive window around the correlation peak is applied that limits the number of samples at the input of the estimation process to increase the robustness of the algorithm to noise. After estimating the CIR, the proposed algorithm estimates the LOS time of arrival from the position of its first non-zero element that passes a certain threshold. Moreover, the phase offsets are computed from the real and imaginary parts of the selected element by using an appropriate phase discriminator function. The performance of the proposed algorithm is compared with the conventional POCS and double delta correlator techniques. Simulation results indicate that the modified POCS algorithm considerably outperforms the double delta correlator technique especially when the number of paths is large (e.g. a specular channel with 7 to 10 paths). When the LOS signal does not exist, the proposed algorithm selects the first received secondary path to estimate the code delay.

The performance of the proposed algorithm is tested through simulation and an actual data collection is conducted in an urban environment.

I. INTRODUCTION

Precise alignment between the locally generated PRN codes and the signals arriving from different satellites is the key to accurate range estimation in a Global Navigation Satellite System (GNSS). However, in urban and indoor areas, due to the effect of multipath in the form of superposition of multiple replicas of
the transmitted signal with different code delays and random phases and amplitudes, LOS signals may not be accurately detected (Pahlavan & Krishnamurthy 1998). This introduces a considerable error to the signal code delay estimation.

Although many GNSS related errors can be mitigated by using differential corrections (e.g. Differential GPS), multipath is a phenomenon that depends on the local environment and cannot be removed by differential processing. Therefore, multipath is the dominant error source in DGPS and its effects on the estimation of the signal parameters must be reduced.

The effect of multipath on pseudorange estimation is widely studied in the literature (e.g. Dragunas 2010), nevertheless, multipath is still a major error source in high precision GPS applications (Misra and Enge 2001). Most of the code delay estimation algorithms in the literature are applicable in the presence of a few secondary paths (three or less) (Dragunas 2010) and therefore not suitable for urban and indoor environments where the number of paths is much larger.

Non-parametric techniques such as narrow correlator, and double delta correlator are among the most popular multipath error reduction techniques and have been around for more than 20 years (Van Dierendonck et al 1992). Nevertheless, as shown by Broumandan et al (2008), these techniques are unable to correctly estimate the code delay where the number of multipath components is large (>3) or when some multipath signals are stronger than the LOS signal.

The second class of multipath mitigation techniques are the parametric delay estimation techniques such as the frequency-domain linear prediction method (Yang et al 2005), least-squares fitting methods (which are equivalent to the maximum likelihood estimation if the noise is white Gaussian) (Manickham et al 1994), cepstral techniques (Bian & Last 1994), and the deconvolution-based techniques (Kostic et al 1992). The latter are based on utilizing the Channel Impulse Response (CIR) to estimate the received signal’s code delay and carrier phase. In frequency-domain linear prediction and least-squares fitting algorithms, the number of multipath is assumed a priori. This results in practical issues since the number of paths is a function of the geometry of scatterers and varies with environment. Also, the cepstral method suffers from poor resolution at low values of SNR.

The Projection Onto Convex Sets (POCS) algorithm, which is a deconvolution-based technique, was introduced by Kostic et al (1992). In this technique, the paths are assumed to be isolated in the delay domain, therefore, the presence of multipath components does not affect the timing of the LOS component.

A deconvolution-based method is based on deconvolving the receiver’s matched filter output to obtain an estimate of the channel impulse response from which the multipath parameters are estimated. The main advantages of these algorithms are their high resolution in the separation of closely spaced multipath components and accurate estimation of attenuation and phase factors of each path (Kostic et al 1992). Also, Lohan et al (2009) showed that the deconvolution-based techniques can be modified for the estimation of synchronization parameters of the new BOC (Binary Offset Carrier) and MBOC (Multiplexed BOC) modulated GNSS signals.

In this paper, a modified POCS algorithm is introduced that estimates the channel impulse response using an adaptive threshold (the choice of the threshold is based on the normalized averaged magnitude of the estimated CIR components and is explained in details in section V) to remove the spurious peaks at each iteration. In order to increase the robustness of the algorithm to noise, an adaptive window around the correlation peak is utilized that limits the number of samples before entering to the estimation process. After estimating the CIR, the LOS time of arrival is achieved from the position of its first element that passes a certain threshold. Whenever the LOS signal is blocked, the first arriving path is used for estimating the code delay. The phase offsets can also be computed from the real and imaginary parts of the selected element by using an appropriate phase discriminator function.

To verify the theoretical predictions, a GNSS data collection is conducted and the performance of the algorithm is compared with the conventional POCS and double-delta correlator techniques.

The remainder of this paper is organized as follows. In section II, the assumed signal model is discussed. Section III gives the formulation of the problem as a system of linear equations. In section IV the least-squares and MMSE estimation algorithms are explained. Simulation and experimental test results are presented in sections VI and VII respectively. Finally, the conclusions are given in section VIII.
II. SIGNAL MODEL

The channel impulse response model in a specular multipath environment can be represented by

\[ h(t) = \sum_{k=1}^{N} \alpha_k \delta(t - t_k) \]  

where \( N \) is the number of paths, \( \alpha_k (|\alpha_k| \leq 1) \) is the complex attenuation factor of the \( k \)-th path which is assumed to be a complex Gaussian distributed random variable and \( t_k, k = 1, ..., N \) is the signal arrival time of the \( k \)-th path. Considering this model, the received signal, which is the output of the multipath channel in response to the transmitted signal \( s(t) \), can be determined as

\[ r(t) = h(t) \otimes s(t) + n(t) = \sum_{k=1}^{N} \alpha_k s(t - t_k) + n(t) \]  

where \( n(t) \) is the channel additive white Gaussian noise (AWGN) and \( \otimes \) denotes the convolution operation. Here, the problem of multipath estimation refers to the estimation of the parameters \( \alpha_k \) (amplitude and phase) and \( t_k \).

III. PROBLEM FORMULATION

Prior to despreading, the received GPS C/A signal power is below the noise floor. Considering the model in Eq. (2) for the received composite baseband signal, the output \( y(\tau) \) of the correlator (matched filter) can be expressed as the convolution of the ideal autocorrelation function of the PRN code, \( g(\tau) \), with the channel impulse response being

\[ y(\tau) = g(\tau) \otimes h(\tau) = \sum_{k=1}^{N} g(\tau - t_k) h_k + v(\tau), \]  

where \( g(\tau) \) is

\[ g(\tau) = \frac{1}{T} \int_{-T/2}^{T/2} s(t) s(t-\tau) dt, \]  

and \( T \) is the code period, and \( v(\tau) \) denotes the noise component at the output of the integration process. Eq. (3) can be written in matrix form as

\[ y = G h + v \]  

where \( y = [y_1, ..., y_M]^T \) is the vector of samples of the matched filter output and \( G \) is

\[ G = \begin{bmatrix} g(\tau_1 - t_1) & \cdots & g(\tau_1 - t_N) \\ \vdots & \ddots & \vdots \\ g(\tau_M - t_1) & \cdots & g(\tau_M - t_N) \end{bmatrix}, \]  

where \( h = [h_1, ..., h_N] \) is the true channel vector with \( h_i = a_i e^{j\phi_i} \) if \( t_i \) is one of the true path delays and zero elsewhere. The vector \( v \) contains the samples of the complex AWG noise term at the correlator output.

Since the autocorrelation function of the transmitted PRN code is a priori known, it is possible to deconvolve the output of the matched filter to estimate the channel impulse response.

IV. MULTIPATH DECOMPOSITION

One of the most well-known techniques for estimating the channel vector \( h \) is the least-squares (LS) solution which is based on minimizing the square error \( \|y - Gh\|^2 \) as

\[ \hat{h}_{LS} = (G^T G)^{-1} G^T y, \]  

where the superscript \( T \) denotes a matrix transpose. It is also possible to estimate \( h \) by minimizing the mean squared error (MSE) as

\[ \hat{h}_{MMSE} = \left( \tilde{\sigma}^2 I + G^T G \right)^{-1} G^T y, \]  

where \( \tilde{\sigma}^2 \) denotes the estimated noise variance.

The projection onto convex sets (POCS) approach is an iterative constraint-based deconvolution approach which estimates the channel impulse response by iterating Eq. (8) as

\[ \hat{h}_{POCS}^{(i)} = \hat{h}_{POCS}^{(i-1)} + \left( \tilde{\sigma}^2 I + G^T G \right)^{-1} G^T \left( y - G \hat{h}_{POCS}^{(i-1)} \right) \]  

Next, it finds the projection of the estimated vector on a set of constraints at each iteration (Kostic et al 1992). In Eq. (9) \( \hat{h}_{POCS}^{(i)} \) is the estimated CIR at \( i \)-th iteration. For initialization, Eq. (8) is used. The required number of iterations depends on the SNR level. At lower SNRs larger number of iterations is required. In practice, iterations are performed until no
significant improvement from iteration to iteration is achieved.

Constraints are generally prior information about the conditions of the problem. Each constraint applies a closed convex set and POCS finds the estimate that complies with all the sets. In other words, if $P$ is the operation of projecting the solution $\hat{h}$ onto a convex set $C$, POCS finds the estimate of $\hat{h}$ by

$$\hat{h}_{POCS} = P\left[\hat{h}\right].$$

(10)

For example, the constraints can be limitations on the variance of the estimation error or the range of acceptable values of delays or anything related to the physics of the problem. Figure 1 shows a block diagram of the receiver implementing the POCS method.

It should be noted here that since the sampling rate of the receiver is limited, the estimated channel impulse response could not be of the ideal form of Eq. (1). For a practical implementation with a limited bandwidth, the estimated CIR can be expressed in more general terms by (Yang & Porter 2005)

$$\hat{h}(t) = \sum_{k=0}^{N} \tilde{\alpha}_k f_s \left[ \frac{\sin(\pi(t-i_k)f_s)}{\pi(t-i_k)f_s} \right]$$

(11)

where $f_s$ is the sampling rate. Figure 2 and Figure 3 show the magnitude and phase of the estimated channel impulse response based on the POCS algorithm respectively. As shown, some of the secondary paths are 180 degrees phase shifted and are causing a destructive effect to the matched filter output. However, POCS is able to estimate individual paths and estimate the magnitude and phase parameters correctly.

The LOS code delay for all of the LS, MMSE and POCS methods is the one corresponding to the maximum magnitude of the estimated channel impulse response vector and is given by

$$\hat{\tau}_{LS/MMSE/POCS}^L = \arg \max_\tau |\hat{h}_{LS/MMSE/POCS}|.$$ (12)

After the LOS delay has been determined, the LOS phase offset is computed from the corresponding complex channel coefficient by applying a proper phase discriminator (e.g., inverse tangent). However, in some applications, it is also desirable to mitigate the effect of multipath from the correlation output. To do this, an equalization block is also added to the system. The equalization block first computes the equalization filter coefficients from the estimated channel response by

$$f = \left\{ \sum_{i=-K/2}^{K/2} \hat{h}_{POCS}^i \left[ \hat{h}_{POCS}^i \right]^T + \frac{\sigma^2}{s} I \right\}^{1/2} \hat{h}_{POCS}.$$ (13)

where $\hat{h}_{POCS}^i$ is $\hat{h}_{POCS}$ shifted by $i$ steps, $K$ is the number of filter taps and $s$ is a constant. Then, it convolves the estimated filter coefficients vector to the matched filter output to remove the effect of multipath.

Figure 1: POCS method Receiver Implementation
V. MODIFIED POCS

The differences between the proposed modified POCS and the conventional POCS can be categorized as follows:

1) In the modified POCS algorithm, the output of the matched filter, $y$, is windowed with a window of the length of only two chips around the largest peak of the autocorrelation function. This decreases the amount of noise that enters the estimation process. Also, only the multipath signals with code delays in the range of a half chip before and after the main peak of the autocorrelation function are determined.

2) In the modified POCS algorithm, an adaptive threshold is applied on the estimated channel impulse response to remove any spurious peaks. This threshold is set as $K_a\bar{a}$ or $K_{MLCR}\bar{a}_{MLCR}$ where

$$\bar{a} = \frac{1}{N} \sum_{k=1}^{N} |h_{POCS}(k)| / \max\{h_{POCS}(k)\} \quad (14)$$

and $a_{MLCR}$ is the magnitude corresponding to the maximum level crossing rate. $K_a, K_{MLCR} > 1$ are two constants that are set experimentally. Thus, the components of the estimated channel impulse response with the normalized magnitudes smaller than the determined threshold are set to zero at each iteration. The simulations results show that the threshold achieved by the first way is more robust. Therefore, the threshold is set as

$$\gamma = \min\{K_a\bar{a}, 1\}. \quad (15)$$

In all the simulations and empirical test results presented in this paper, $K_a$ was set to 2.5.

3) In the previous section it was shown that the accuracy of the POCS algorithm is determined by one sample. In other words, the accuracy is limited by the receiver’s sampling rate. However, the achievable sampling rates are limited in practice. Therefore, some of the true channel components may not be located on the samples taken from the estimated channel. When a channel component is not located on the samples taken, the samples on both sides will take relatively large values that indicate that a true channel component is located at that area. Figure 4 depicts such a situation.
In order to increase the accuracy of the modified POCS algorithm, a linear interpolation is applied between the samples around the detected LOS as

\[
\tau = (k_L - 1) + \frac{|h(k_L)| + |h(k_L + 1)|}{2} \text{ samples} \quad (16-a)
\]

and if \( |h(k_L - 1)| < |h(k_L + 1)| \)

\[
\tau = k_L + \frac{|h(k_L + 1)|}{|h(k_L + 1)| + |h(k_L)|} \text{ samples} \quad (16-b)
\]

where \( h_{\text{ave}} \) is the estimated channel impulse response non-coherently averaged over successive epochs and \( k_L \) is the index corresponding to the detected LOS.

4) In the modified POCs method, the code delay of the LOS signal is estimated as the one corresponding to the first component of the vector \( \hat{h}_{\text{POCS}} \), its normalized magnitude being above the predetermined threshold.

VI. SIMULATION RESULTS

In order to verify the performance of the proposed algorithm, a few simulation scenarios were performed using a Sprient GSS 7700 hardware simulator. The RF signals from the hardware simulator were passed through an LNA to a front-end that outputs raw IF I/Q pair samples at the sampling rate of 25 MHz. In these simulations, a multipath channel profile with a LOS path and seven secondary paths was considered. The estimated channel impulse response for the hardware simulator data is depicted in Figure 5. In the simulation scenario related to this figure, the path attenuation factors were randomly selected from a range of 0 to 6 dB. It can be observed that the estimated channel parameters by the modified POCS method match the true values.

In Figure 6 and Figure 7, the performance of some different estimators in a 4-path channel profile is compared. The correlator spacing for the double-delta correlator is around 0.07 of a chip.
It is observed from these figures that at low values of $C/N_0 (<20 \text{ dB})$, the modified POCS algorithm considerably outperforms the conventional POCS estimator and the conventional POCS outperforms the LS estimator. For the case of this simulation, the maximum magnitude of the autocorrelation function was close to the true LOS. In this case, as it can be observed from Figure 6, the double-delta correlator could slightly improve the estimation of the code delay corresponding to the autocorrelation function (ACF) peak.

In the simulation scenario related to Figure 8 and Figure 9, an 8-path multipath channel profile was considered.

Consequently, there is a large bias in the estimation of conventional autocorrelation function based method and the double delta correlator has even increased this error. However, the performance of the POCS and modified POCS algorithms has not changed significantly. Therefore, since the number of paths was not small and some of the secondary paths were stronger than the LOS path, the double delta correlator technique was unable to correctly estimate the code delay.

VII. EXPERIMENTAL ASSESSMENT

A data collection was performed to test the proposed algorithm with real data. The estimated pseudoranges computed by different algorithms and the required ephemeris information were fed to the position solution unit. The test was performed in a typical urban location close to the Calgary Center of Innovative Technology (CCIT) building (Figure 10). In order to cancel the effect of other error sources other than multipath, a differential GPS scenario was utilized. The reference antenna was located on the roof of the CCIT building and the rover antenna (a Ublox antenna) was located on the ground and was surrounded by some other buildings. A dual-channel National Instruments (NI) front-end was used for collecting synchronized signals from reference and rover antennas. The RF signals were passed through LNAs to the NI front-end and were sampled (IF I/Q pair samples) at the rate of 25 MHz.

In case of this simulation, the maximum magnitude of the autocorrelation function was about 0.7 of a chip delayed with respect to the true LOS.

While for the reference antenna there were 11 to 12 visible satellites, the rover antenna had only four visible satellites in view (PRNs 20, 25, 30 & 31). The true position of the rover antenna was surveyed by a
The position of the reference antenna was known a priori.

The estimated CIRs by the modified POCS algorithm showed that there were several strong secondary paths close to the estimated LOS for PRNs 20 and 25. Moreover, for PRN 31, a phase reversal occurred at the estimated position for the LOS path. Consequently, the ACF peak was shifted by about 0.15 of a chip while for PRN 30 the ACF peak was shifted less than 0.06 of a chip. Figure 11 shows the estimated CIR for PRN 31. The plots in Figure 12 show the reference-rover pseudorange difference errors for the related PRNs using both of the conventional and modified POCS algorithms and Table 1 tabulates the mean and the standard deviation values of the estimation errors. The correlator spacing for the double-delta correlator was set to 0.8 of a chip.

![Figure 11: Estimated CIR for PRN 31](image1)

![Figure 12: Reference-Rover pseudorange difference errors computed by conventional and modified POCS algorithms](image2)
Table 1: Values of biases and standard deviations of estimated pseudorange differences

<table>
<thead>
<tr>
<th>PRN</th>
<th>POCS Bias [m]</th>
<th>Conv. Algorithm Bias [m]</th>
<th>POCS Std. [m]</th>
<th>Conv. Algorithm Std. [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>3.1</td>
<td>9.3</td>
<td>3.5</td>
<td>6.6</td>
</tr>
<tr>
<td>25</td>
<td>-8.4</td>
<td>39.5</td>
<td>4.1</td>
<td>7.1</td>
</tr>
<tr>
<td>30</td>
<td>-4.7</td>
<td>-16.7</td>
<td>2.6</td>
<td>5.5</td>
</tr>
<tr>
<td>31</td>
<td>-9.4</td>
<td>-37.3</td>
<td>5.2</td>
<td>6.7</td>
</tr>
</tbody>
</table>

The reference-rover pseudorange difference errors computed by the conventional ACF-based double delta correlator algorithm have large biases around 40 metres for PRNs 25 and 31 while for PRNs 20 and 30, the values of bias were smaller. Also the pseudorange differences computed by the modified POCS algorithm has a considerable bias of 9.4 m for PRN 31 caused by the spurious peaks resulted from noise and the possible aliasing effect of insufficient sampling rate.

Figure 13 shows the position results for this data set. These results are computed by applying the least-squares solution to the pseudorange differences computed by both the modified POCS and the double delta correlator methods. The mean and the standard deviation values of the estimated positioning errors are summarized in Table 2.

As Table 2 shows, compared to the double delta correlator technique, applying the modified POCS algorithm has almost considerably decreased the values of bias in the computed positioning solutions. Also, it is observed that the values of standard deviation for the solution computed by the modified POCS algorithm are slightly smaller than the conventional algorithm.

Table 2: Values of biases and standard deviations of the computed positioning errors

<table>
<thead>
<tr>
<th></th>
<th>POCS Bias [m]</th>
<th>Conv. ACF-Based Algorithm Bias [m]</th>
<th>POCS Std. [m]</th>
<th>Conv. ACF-Based Algorithm Std. [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eastward Error</td>
<td>-10.3</td>
<td>27</td>
<td>5.7</td>
<td>10.6</td>
</tr>
<tr>
<td>Northward Error</td>
<td>8.2</td>
<td>-43.4</td>
<td>10.8</td>
<td>17.9</td>
</tr>
<tr>
<td>Upward Error</td>
<td>1.8</td>
<td>32.2</td>
<td>2.1</td>
<td>2.7</td>
</tr>
</tbody>
</table>

Figure 13: Position errors
I. CONCLUSIONS

A modified POCS algorithm for estimating the code delay and the carrier phase of the GNSS signals with only a few iterations was introduced. Simulation results showed that the modified POCS algorithm outperforms the conventional POCS at low SNR. It was also shown that the double delta correlator technique fails to correctly estimate the LOS code delay when the number of multipath components is large or when some multipath signals are stronger than the LOS signal.

The field test results also verified that compared to the conventional ACF-based techniques, the modified POCS algorithm considerably decreases the error bias in the position solution. These results, obtained in controlled simulated environments, should be further verified by actual tests in urban canyons.

REFERENCES


