An Enhanced Two-Step Least Squared Approach for TDOA/AOA Wireless Location

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Abstract- An Enhanced Two-Step Least Squared (LS) approach is proposed here for hybrid Time Difference of Arrival/Angle of Arrival (TDOA/ AOA) wireless location. Compared to the original two-step LS algorithm, the method here can provide better performance, almost the same as that of Taylor-Series estimator. Compared to the Taylor-Series solution, the method has many computational advantages, such as, light computational burden, no linearization, no initial point selection issue, and no convergence issue.

I. INTRODUCTION

Wireless location determines the position of a mobile station (MS) in a wireless communications system. It has received considerable attention over the past a few years [1] because of wide applications, such as, emergency 911 (E-911) subscriber safety services. Various wireless location schemes have been extensively investigated [2,3]. In this paper, two categories of schemes, time based scheme and angle based scheme, are paid particular attention to since they are complementary to each other. The location estimator, based on the distance measurements and direction measurements are usually nonlinear. In [4], the solution to the TDOA equations is obtained by linearizing the equations via Taylor-series expansion. In [5], a so-called two-step LS estimator for TDOA location is proposed. In the first step, an intermediate MS location is calculated under the assumption that MS location is independent on the distance between the MS and a reference base station (BS). But, the distance and MS location is actually correlated, so the intermediate result needs to be adjusted in the second step to get a better final result.

However, the original two-step LS approach is not optimal. This is because the second step does not take into account any measurement information which should also constrain the final solution, thus resulting in inferior accuracy. In this paper, a new method, Enhanced Two-Step LS, is proposed to take care of this deficiency. By approximating the relationship between the MS and the reference station with a linear model around the intermediate result, this approach makes full use of measurements to adjust the intermediate result to provide better performance. The organization of the paper is as follows. After this introduction, the TDOA/AOA models are discussed in section II. Section III describes the Enhanced Two-step LS approach in details. Simulation results in section IV demonstrate the performance improvement. Finally, conclusions are drawn.

II. TDOA/ AOA WIRELESS LOCATION

A. TDOA-Based Wireless Location

The TDOA system determines the MS position based on trilateration technique, as shown in Figure 1. It is often referred to as a hyperbolic system because time differences are equivalent to distance differences that form hyperbolic curves. The TDOA scheme is a non-linear problem. It tries to solve the following optimization problem

$$\hat{x} = \arg \min_x \sum_{i,j,k \in S} \left( r_{ij} - \|x - X_i\| - \|x - X_j\| \right)^2$$

(1)

where $x$ is MS location, $r_{ij}$ is the range difference measurement of the MS to the $i$-th and $j$-th BSs, $S$ is the set of all BSs, and $X_i$ and $X_j$ are coordinates of BS$i$ and BS$j$.

B. AOA-Based Wireless Location

Signal AOA information, measured at BSs with an antenna array, can be used for positioning purpose as in Figure 2.

MS is at the intersection of several direction lines corresponding to AOA measurements. An AOA system...
normally tries to determine the MS location by solving the following problem
\[
x = \arg \min_{x \in \mathbb{R}^n} \sum_{i=1}^{n} \text{dist}(x, \beta_i)^2
\]
\[
\text{dist}(x, \beta_i) = \|x - y_i\| - \sin \beta_i (x - x_i) + \cos \beta_i (y - y_i)
\]
where \(\beta_i\) is the measured direction angle or direction line between MS and BS, \(d_{\text{dist}}\) is the distance between the calculated MS and the measured direction line \(\beta_i\).

**C. Hybrid TDOA/AOA Solution**

To improve positioning accuracy, it is better to use as much information as possible. A hybrid solution is proposed by simply combining TDOA and AOA measurements as follows.

\[
\begin{bmatrix}
x_2 - x_1 & y_2 - y_1 & r_{21} \\
\vdots & \vdots & \vdots \\
x_n - x_1 & y_n - y_1 & r_{n1} \\
\sin \beta_1 & -\cos \beta_1 & 0 \\
\vdots & \vdots & \vdots \\
\sin \beta_n & -\cos \beta_n & 0
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
r_1 \\
\sin \beta_1, x_1 - \cos \beta_1 y_1 \\
\vdots \\
\sin \beta_n, x_n - \cos \beta_n y_n
\end{bmatrix}
= \begin{bmatrix}
(K_2 - K_1 - r_{21}^2)/2 \\
(K_n - K_1 - r_{n1}^2)/2 \\
\sin \beta_1, x_1 - \cos \beta_1 y_1 \\
\vdots \\
\sin \beta_n, x_n - \cos \beta_n y_n
\end{bmatrix}
\]

Where \(\beta_i\) is the AOA of MS signal with respect to BS, \(r_i^2 = (x_i - x)^2 + (y_i - y)^2\), \(K_i = x_i^2 + y_i^2\), and \(r_{11} = r_1 - r_i\). A so-called Two-Step LS method can be used here to estimate the MS location. In the first step, it gets an intermediate estimation of \(x, y, r_1\) from the linear subsystem (3) under the assumption that they are three independent variables. In the second step, this intermediate result is further adjusted to fit the following relationship
\[
r_i^2 = (x_i - x)^2 + (y_i - y)^2
\]
among \(x, y, r_1\) by solving an artificially created linear equation. More information about this two-step LS approach can be found in [5].

**III. ENHANCED TWO-STEP LS APPROACH FOR TDOA/AOA WIRELESS LOCATION**

The original Two-Step LS approach in [5] for TDOA/AOA hybrid problem is not optimal because the second step takes only the relationship (4) into account. It does not use any information that appeared in the first step (3). But, the information actually also constrain the final result. Such deficiency will deteriorate the positioning accuracy.

The proposed enhanced two-step LS solution uses another method in the second step which can overcome this problem. It first studies all measurements in (3) and equation (4) in a 3D space, then identifies and approximates the non-linear part with a linear one to reduce the problem, the combination of (3) and (4), into a true linear problem, and finally the normal LS method is applied to give the better final result.

If \(x, y, r_1\) are recognized as three independent variables of a 3D space, the equations in (3) are a set of planes. So, the subsystem of equations (3) is a linear subsystem. Unfortunately, equation (4) has squared calculation in it meaning the relationship among \(x, y, r_1\) is nonlinear. In fact, equation (4) stands for a cone in 3D space as shown in Figure 3.

![Fig. 3. Relationship Among \(x, y, r_1\)](image)

From the above plot, one can easily see that a small region of the cone is quite like a plane especially when the MS is far from the reference base station, BS1. So, if the cone is approximated near the intermediate result with a plane, the whole system, the combination of (3) and (4), can be reduced into a true linear system. LS can then be easily used to get the final result. In this case, we can use not only the relationship among \(x, y, r_1\) but also all measurement information appeared in the first step. It is reasonable to expect an improved performance.

The idea behind the approximation is straightforward: First select an initial point surrounding by an interested region on the cone based on the intermediate result, secondly approximate this region with a plane passing through this initial point, and finally adjust this plane to fit the interested region best.

Supposing the intermediate result is \((x', y', r_1')\), the linearization can be made around \((x', y')\) where the MS is supposed to be. For simplicity, the coordinates of the reference base station, BS1, are supposed to be \(x_1 = 0\) and \(y_1 = 0\) in the succeeding discussion. Thus, the initial point corresponding to \((x', y', r_1')\) on the cone can be chosen as \(P = (x', y', \sqrt{(x')^2 + (y')^2})\). The best plane passing P to approximate the cone is the one that is tangent to the cone. To find it, we need the normal vector of the cone passing point P. From the relationship among \(x, y, r_1\), one can easily find that this normal vector is \(-x, -y, \sqrt{(x')^2 + (y')^2}\). Thus the plane we are searching for is:

\[
-x' (x - x') - y' (y - y') + \sqrt{x'^2 + y'^2} r_1 = 0
\]

However, this plane is in fact not optimal because the distances from the points on the interested cone region to this plane are not minimized. To get a strictly optimal result, the following problem needs to be solved
\[
\text{plane} = \arg\min \int_S \text{dist}(\text{Point}_{\text{cone}}(s), \text{plane}) \, ds
\]
where \( S \) denotes the interested cone region. Unfortunately, it is not easy to solve this equation. For simplicity, a suboptimal solution is proposed instead by adjusting the plane (5) with respect to the interested cone region to minimize the difference as shown in Figure 4.

\[x_p = \sqrt{x'^2 + y'^2 + \sigma^2} \cos(\theta \pm \Delta \theta)\]
\[y_p = \sqrt{x'^2 + y'^2 + \sigma^2} \sin(\theta \pm \Delta \theta)\]
\[r_i = \sqrt{x'^2 + y'^2 + \sigma^2}\]

where
\[
\theta = \tan^{-1} \left( \frac{y'}{x'} \right) \quad \Delta \theta = \tan^{-1} \left( \frac{\sigma}{\sqrt{x'^2 + y'^2}} \right)
\]

The maximum distance is
\[
d_{\text{max}} = \left| -x'x_p - y'y_p + \sqrt{x'^2 + y'^2} r_i + D \right| = \frac{d_{\text{max}}}{2}
\]

Thus
\[
2\left| -x'x_p - y'y_p + \sqrt{x'^2 + y'^2} r_i + D \right| = \left| -x'x_p - y'y_p + \sqrt{x'^2 + y'^2} r_i + \sqrt{x'^2 + y'^2} \right|
\]

\( D \) can be easily determined from this equation.

After the approximation, the combination of (3) and (4) becomes a true linear system. MS location can be readily solved from a constrained LS problem (3) with constraint
\[-x'x - y'y + \sqrt{x'^2 + y'^2} r_i + D = 0\]

IV. SIMULATION RESULTS

This section presents simulation results to demonstrate the performance improvement of the enhanced Two-Step LS approach compared to the original method and Taylor-Series method. In the simulation, a 7-cell 2D cellular phone system layout is assumed as shown in Figure 5. Furthermore, we also assume the MS is in the center hexagonal cell BS1 with six adjacent hexagonal cells of the same size around it. The cell radius is set to be 2000m. For simplicity, we assume that all TDOA measurement noise has the same standard deviation. All experiments here are Mont-Carlo experiments and each experiment contains 1000 independent runs.

A. Algorithms for Comparison

In this section, the performance comparison among the Taylor-Series method, Original Two-Step LS method, and Enhanced Two-Step method will be conducted. We also investigate the performance improvement from TDOA only methods to hybrid TDOA/AOA methods and the influence of AOA measurement accuracy on the final location performance. So the algorithms studied here are:

TDOA Only Taylor Series

Original TDOA Only Two Step LS

Enhanced TDOA Only Two Step LS

The CEP errors of the original TDOA only Two-Step LS method and the original hybrid TDOA/AOA Two-Step LS method are about 150m. The CEP errors of the TDOA only Taylor-Series solution and TDOA only Enhanced Two-Step LS solution are both around 55-60m. The CEP errors of the hybrid TDOA/AOA Taylor-Series solution and hybrid TDOA/AOA Enhanced Two-Step LS solution are about 35-40m. This means that the performance of the original Two-Step LS is the worst. The performance of the Enhanced Two-Step LS approach is almost as accurate as the Taylor-Series approach while still maintaining the computational advantages of the original Two-Step LS method. With the combination of TDOA and AOA measurements, the positioning accuracy is significantly improved.

Figure 6 shows the performance comparison among these six algorithms. In this scenario, we assume that only 4 TDOA measurements and 2 AOA measurements are available, that the standard deviation of the TDOA measurement noise is 100m, that the standard deviation of the AOA measurement noise is 1 degree, and that the position of the MS is (1500m, 1500m). Two methods to evaluate the performance are used. One is shown in Figure 6(a) where we study the Root-Mean-Square of positioning error with respect to the probability that the position error is smaller than an error threshold. The horizontal axis is the probability that position error is smaller than a threshold. The vertical axis is the RMS of positioning error. Obviously the lower the plot is, the better the performance is. The second method is shown in Figure 6(b) where we study the cumulative probability with respect to a position error threshold. The horizontal axis is position error threshold and the vertical axis is the cumulative probability - that the positioning error is smaller than the corresponding error threshold. Obviously, the higher the plot is the better the performance. Actually, these two methods are equivalent and either may be used.

Figure 7 shows the performance comparison when the standard deviation of the TDOA measurement noise is increased to 300m while maintaining the other parameters the same as that of the first experiment. In this case, the CEP errors of the two original Two-Step LS methods exceed 200~250m. The CEP errors of the other two TDOA only solutions are about 150m, and the CEP errors of the other two hybrid TDOA/AOA solutions are around 60m. We get similar results as those in the first experiment. The two original Two-Step LS algorithms are the worst, the enhanced Two-Step LS methods are almost as good as the Taylor-Series solutions, and the combination of AOA with TDOA can improve positioning accuracy. But, the accuracy decreases compared to that of the first experiment because of a much larger TDOA measurement noise.
To check the influence of the AOA measurement accuracy on the final positioning accuracy, two experiments were performed. In the first experiment shown in Figure 8, the standard deviation of AOA measurements is 5 degrees, and we can see there is almost no performance improvement when such AOA information is used. This is because the accuracy of AOA is so bad that it cannot provide any useful information to the system. In the second experiment shown in Figure 9, the standard deviation of AOA measurements is 0.3 degree although it is a little bit difficult to measure AOA with such a high accuracy. We can see that performance is significantly improved when accurate AOA measurements are used. These two experiments demonstrate the importance of measuring AOA accurately in a hybrid positioning method.

V. CONCLUSIONS

In this paper an Enhanced Two-Step LS approach was proposed for hybrid TDOA/AOA wireless location. Compared to the original two-step LS algorithm, the method herein can provide better performance and is almost as accurate as a Taylor-Series estimator. The reason is that, unlike the original two-step method, the second step of this enhanced method makes use of all measurements. Compared to the Taylor-Series solution, the method has many advantages, such as, light computational burden and no convergence issues. The hybrid solution can improve performance by integrating AOA measurements. However, the improved performance can only be obtained when the AOA measurement accuracy is sufficiently high.

REFERENCES